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CHAPTER 1

TEACHING MATHEMATICS

THIS CHAPTER

- examines the typical motivations and anxieties of mathematics trainee teachers as they begin training
- summarizes the professional demands of training
- discusses different perceptions of the nature of mathematics; whether it is primarily a subject that is interesting in its own right, or whether it is primarily a tool for solving problems
- considers why people think that mathematics is important
- examines the extent to which mathematics is a human construct

Your early days as a trainee teacher

Training to be a teacher is a challenging and demanding task, but one that provides enormous rewards. One of the biggest rewards is that moment when a pupil's eyes light up with a sense of understanding, and you know that a small piece of learning is down to you. During your training, you will feel the satisfaction of knowing that your professional skills are developing, and will have the opportunity to work alongside teams of dedicated staff, whose energy you will come to admire.

It is common for people embarking on a teacher-training course to have a number of anxieties. One of these anxieties often concerns subject knowledge. Many people starting training have come straight from university, and many come after a break from study of several years. Whichever background you have, you are likely to be 'rusty' on some elements of school mathematics, either because of the passage of time, or because the mathematical topics you studied at university were very abstract or specialized and did not require the use of topics from a school curriculum.

You are very likely to meet others like you: maybe other mature students, others whose first degree is not mathematics or others who may never have studied any mechanics. People come from a wide range of backgrounds wanting to become mathematics teachers.

During these early days, you may have some other anxieties, such as whether you will be able to control the class or whether you will cope with the workload. Such worries are common, but they can be answered:

- You will get lots of support from other teachers.
- You will not be given the worst class in the school.
- You will learn techniques of class management.
- Workload will be heavy, but it is manageable.

It is important in the early days to get to know others who are training alongside you: they will provide invaluable support as you progress through your training. Not only will they share with you their highs and lows, they will also be able to share ideas for planning lessons. If, additionally, they are training to teach mathematics, an agreement to share resources can save you time and effort.

As soon as your training begins, you will be asked to focus on the standards by which you will be judged during, and at the end of, the training. These standards are set nationally and are known as the Teachers' Standards (DfE, 2013). The Standards document is divided into three parts: the Preamble, which explains the purpose of the standards; Part One, which consists of eight standards for teaching with further amplification of each standard; and Part Two, which outlines the standards for personal and professional conduct. In order to qualify as a teacher you will have to provide evidence that your practice is consistent with the principles outlined in the Preamble and that you meet each of the standards listed in Part One. A listing of the Standards is given in the 'How to use this book' section at the beginning of this book. Your training is designed to help you meet the standards.

One of the best ways to feel your way into the training is to develop your knowledge of the school mathematics curriculum. Whether directed to or not, it is helpful to get hold of a recent GCSE paper and work through it. The best exercise is for you to work through it as part of a group, discussing the bits that you have forgotten and explaining to others the bits that you can remember. It will not take long for your confidence to return, but make sure you check that the answers that you think are right are indeed the correct ones! Sometimes your confidence may be misplaced, and it is better to discover and correct your own misunderstandings in supportive company than when you are in the classroom.

Professionalism

During your training, high standards of attendance, punctuality and commitment are expected. When you are in school, you need to be aware of the behaviour and standards expected from teachers, and you should exhibit those standards from day one. Although you may not feel like a teacher on your first day, you need to behave like one: the pupils will observe the way that you are and will be forming impressions of you! In addition to the pupils, the teaching staff will be forming impressions of you and, obviously, it is important that you give the right message through the way you work and behave. Here are a few key pointers to acceptable behaviour in school:

- Arrive in plenty of time each morning; be punctual to lessons and meetings.
- Always let the school know if you are going to be absent or late.
- Treat all members of the school staff with respect. You need to demonstrate that you can work well with others.
- Dress smartly. (Play safe on day one, and after that take your cue from other members of the department.)
- Treat the pupils with respect. Show concern for their learning and welfare, but do not become over-friendly (especially when you are new to the school). Begin to build positive relationships, where pupils see you as someone to be trusted, someone who is fair and someone who can help them to achieve well.
- Set a good example to the pupils. This extends beyond your appearance and behaviour, and includes your values and attitudes; you should demonstrate positive attitudes and encourage the same in your pupils.
- Remain in school during the whole of the normal school day. In general, you should not leave the premises except at lunchtime.
- Use productively any non-contact time that you have. Do not be seen wasting time in the staff room; teachers are busy people, and they will expect to see you working

as hard as they are. Do not fall into the trap of believing that your working day ends when teaching ends.
- Give positive support to colleagues, especially within the classroom.
- Take care to learn the school rules. Once you are part of the school staff (even as a trainee) you join in the collective responsibility for the implementation of school policies. This professional responsibility supersedes your personal opinion, so you must promote, for example, policies on school uniform, even if you disagree with pupils having to wear a uniform.

In each school where you work, you will be assigned a mentor within the mathematics department. Mentors are key people in your training; they will provide support and advice, and will also make judgements on you. You need to demonstrate your professionalism to all staff, but particularly to your mentor, throughout your time in school. For example, you can show your commitment to team-working through offering any original work you have done to the rest of the department. (Departments are very likely to have facilities for electronic sharing of worksheets, presentations and links.)

Above all, you need to be aware that during your time in schools, it is not only your skills as a teacher that are being assessed, but also your professional attributes. Part Two of the Teachers' Standards document is devoted to personal and professional conduct. There are three main headings with some further subdivisions giving eight statements which define the expectations. Having positive attitudes yourself helps, but you need to go beyond this; you must make it clear to everyone (pupils, teachers and, on occasions, parents) in all that you say and do, that you have high expectations and are committed to helping pupils to achieve their very best.

Motivations

The motivations for wanting to teach are many and varied. Some of the most common reasons for starting to train as a mathematics teacher are:

- to pass on my enthusiasm for the subject
- to make a positive difference
- to do something worthwhile
- it is something that I expect to enjoy/find rewarding (Chambers, 2007).

Many potential teachers mention their enjoyment of doing mathematics. Sometimes they have a fascination for mathematics, and a curiosity that they wish to share with others; in other cases their love of mathematics is based on little more than a personal experience of success in the subject. Examples of trainees' motivations are given in the following quotes:

I want to turn children on to mathematics, in the same way that I am.

I had one particular mathematics teacher who I really admired, and she made all the difference to me. I would like to be that inspiring teacher to others.

I love mathematics and yet it has such a negative image. I want to help change that image.

I want to show children that mathematics is a fascinating subject.

Many people give their reason for wanting to teach as a desire to do something worthwhile, to feel that their efforts can make a difference to the lives of young people. Sometimes this follows many years spent working in industry, where the main motivation of the workforce is to make money. In comparison with this, teaching is seen as contributing to the common good and to the benefit of society.

Others mention their desire to work with young people. Many cite an experience that has helped confirm their decision. This experience may involve one or more of the following:

- helping out a friend or colleague with their mathematics
- coaching a child in school mathematics (possibly in preparation for an examination)
- working with youth groups (such as scouts, guides, church youth groups)
- observation of school mathematics lessons.

In practice, all training routes will expect applicants to have spent some days in a secondary school, observing mathematics lessons before being accepted for training. There is little doubt that this gives the best insight into whether mathematics teaching is something that will provide a suitable career.

The most common anxiety for those starting training is 'keeping control of my classes'. This is understandable, particularly in view of press stories that tend to emphasize the sensational and the negative aspects of what goes on in schools. Other common worries include coping with the anticipated workload during training and more general worries about the expected workload of being a teacher in the long term.

Sadly, you may find that, when you tell people about your decision to train as a mathematics teacher, you get replies that are less than encouraging. Reactions tend to be heavily dominated by negative attitudes to mathematics, so that common responses include:

- You must be mad.
- I could never do maths.
- You must be clever.
- I used to hate maths.

This gives some indication, if any more were needed, of the widespread fear and/or dislike of mathematics among the adult population.

POINT FOR REFLECTION

Consider your own motivations for wanting to become a mathematics teacher, and whether they fit in with the most common reasons given above. You have no doubt already discussed your decision to become a mathematics teacher with your family, friends and possibly former work colleagues; what were their reactions? Reflect on why they may have the negative image that they have. Consider why so many adults feel comfortable with (even proud of) their limited ability in mathematics. Was anyone envious of your decision?

List three things that you think you will enjoy about teaching.
List any aspects that you think you might not enjoy.

As you continue to talk to people from outside teaching during your training, note how many of them give a negative response to teaching or to mathematics.

What is mathematics?

Anyone thinking of taking up the career of mathematics teacher needs, at some time, to consider this rather big question. In some ways, it is possible for specialist mathematicians in universities and schools to neglect it. Specialist mathematicians are often consumed by a knowledge of, and enthusiasm for, the subject. They take for granted that they understand what mathematics is, and that mathematics as a subject is a valuable area of study. At higher levels of mathematics, the study becomes so specialized that there is often no need for the bigger picture. Even within schools, there are teachers who have a thorough knowledge of the elements of the subject, but whose appreciation of the subject as a whole is weak.

Many people have tried to describe what mathematics is. Most definitions use words like logical ideas, interconnected ideas, relationships, patterns; some include other aspects such as communication, or particular sub-sections like the appreciation of number. Many discussions of the nature of mathematics make distinctions between mathematics as a subject to study in its own right and a subject that is useful. Ernest (1991) characterizes the distinction as deciding which is more important:

- understanding that $5 \times 23 = (4 \times 23) + (1 \times 23)$, or
- understanding that finding the cost of 5 apples at 23p each involves calculating 5×23, and knowing a way of doing it.

This distinction is typical of the different outlooks of pure mathematicians (purist views) and applied mathematicians or engineers (utilitarian views).

The views of the purists can be summarized in the following terms.

Mathematics is:

- objective facts
- a study of reason and logic
- a system of rigour, purity and beauty
- free from societal influences
- self-contained
- interconnected structures.

From the point of view of the extreme purist, applied mathematics is looked down on as being based more on skills than understanding. Applications are inferior to the set of structures that make up pure mathematics, and whether or not a branch of mathematics is useful is an irrelevance. From this point of view, mathematics is a higher-level intellectual exercise, an art form and an example of the creativity of the human mind. Words like aesthetics and elegance are important to the purist (Scopes, 1973).

The description of mathematics as 'what mathematicians do' is sometimes used, but this seems to avoid giving a straight answer. Similarly, to describe the aim of performing mathematical investigations as a means 'to cultivate the art of doing mathematics' (Gardiner, 1987) seems to be insufficient justification. Smith (2004: 11) refers to the value of learning mathematics as being something that 'disciplines the mind, develops logical and critical reasoning, and develops analytical and problem solving skills to a high degree', which is a more helpful articulation of the purist standpoint. 'It must not be imagined', writes Bell (1953: 2), 'that the sole function of mathematics is to serve science ... mathematics has a light and wisdom of its own, and it will richly reward any human being to catch a glimpse of what mathematics means to itself'. Similarly, Pedoe (1958: 9) refers to mathematics as not only being of interest to the science student; the subject is also of interest to those with arts backgrounds, and contains 'much which is beautiful and should interest everyone'.

The mathematics curriculum has tended to reflect the spirit of the times. The 1960s was a time of free expression, experimentation and challenges to authority, and it was purist views of mathematics that drove the curriculum changes that took place in UK mathematics classrooms in those years. Topics such as set theory, number bases and matrices were studied for the first time in the 11–16 curriculum. Transformation geometry replaced traditional Euclidean geometry, and there was more emphasis on probability and statistics. The philosophy behind these changes was that pupils needed the opportunity to see the rich structures in mathematics, whereas the existing curriculum had too much focus on routines and techniques. Many mathematics teachers embraced the changes with enthusiasm, but found it difficult to transfer this enthusiasm to parents, who regarded 'new maths' with suspicion.

In the 1980s, this view of mathematics gradually became replaced by a much more utilitarian view of the subject. The spirit of the times was much more geared towards economic success, and this was reflected in mathematics classrooms. Certainly by the end of the decade, education in general was being heavily influenced by the perceived needs of industry. Thus, applications became the most important part of mathematics, and it was from this point of view that the National Curriculum was first developed. In the utilitarian view of mathematics, learning how to do mathematics can become more important than understanding the underlying principles. Thus mathematics is characterized as:

- a tool for solving problems
- the underpinning of scientific and technological study
- providing ways to model real situations.

By no means does all the pressure for school mathematics to move away from its purist influences come from the political field. Many in the mathematical world share the view that mathematics should be presented as something that is useful, and this view was highly influential in introducing the General Certificate of Secondary Education (GCSE) and the National Curriculum. Hence mathematics began to be taught more through applications and contextual situations, with the aim of increasing pupil motivation through demonstrating relevance (for example, Burkhardt, 1981; Mason et al., 1982). According to this view, mathematics is less about knowing and more about doing. There is an acceptance that pupils should study the pure mathematical skills, but it is the applications that bring the subject to life. This philosophy of teaching mathematics is backed up by pointing to the wide range of applications, including the social sciences, biology and medicine, where mathematics makes a contribution (Burghes and Wood, 1984).

The Smith Report (Smith, 2004) opens with several paragraphs about the place of mathematics within the curriculum. But the spirit of the times is perhaps reflected in the fact that only the first paragraph is devoted to a discussion of mathematics for its own sake. There then follow seven further paragraphs devoted to the usefulness of mathematics in a variety of fields – for the knowledge economy, for science, technology and engineering, and for the workplace. Contrast this with the summary of definitions of mathematics given by Orton (1994: 11): 'an organized body of knowledge, an abstract system of ideas, a useful tool, a key to understanding the world, a way of thinking, a deductive system, an intellectual challenge, a language, the purest possible logic, an aesthetic experience, a creation of the human mind', where the utility of the subject is only a minor aspect.

The utilitarian view of mathematics remains a key influence on the curriculum, but it is widely acknowledged that mathematics should be presented as a subject in its own right, a subject that can inspire and challenge at all levels. While successive revisions of the mathematics National Curriculum have continued to promote the idea that mathematics is, above all, useful, the most recent revision takes more note of purist views of the subject than previous versions. For example, the very first paragraph describes mathematics as 'a creative and highly interconnected discipline' and states that studying mathematics provides a foundation for 'an appreciation of the beauty and power of mathematics' and 'a sense of enjoyment and curiosity about the subject' (DfE, 2014). These descriptions are placed alongside statements about its usefulness in solving problems, in science and engineering, and in 'most forms' of employment.

So mathematics is a study of patterns, relationships and rich interconnected ideas (the purist view). It is also a tool for solving problems in a wide range of contexts (the utilitarian view). There is a third common answer to the question of what mathematics is, which says that mathematics is a means of communication. Mathematical language is a wonderful way of communicating ideas, which works across international boundaries, and is not subject to individual interpretations of meaning. Adrian Smith describes mathematics as providing 'a powerful universal language and intellectual toolkit for abstraction, generalisation and synthesis' (Smith, 2004: 11). Using mathematics, we 'convey ideas to each other that words can't handle' (Alison Wolf, quoted in DfEE, 1999a: 15).

Mathematics as a language has many facets. Within the English language, as in others, mathematics uses its own specialist vocabulary which helps to communicate specific ideas in a precise and unambiguous way. Developing this mathematical vocabulary is necessary if pupils are to have access to learning higher levels of mathematics, where specialists will routinely use this vocabulary. The National Curriculum acknowledges the importance of this: 'the quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof' (DfE, 2014).

A particular facet of mathematics as a language is algebra. This is a truly international language and is something that helps to bind the international mathematical community together. The use of Arabic letters for unknowns is universal, even where languages use different scripts. Similarly, symbolic conventions like powers, roots, integrals, and so on, are recognized by the international community, so teaching mathematics helps pupils to have access to this rich body of internationally communicated ideas.

Another dimension to the debate about the nature of mathematics is the extent to which mathematics is a body of knowledge as opposed to a way of working. For example, in a book about mathematical investigations, Gardiner (1987) presents a series of problems that are not in themselves important, but 'what is important is the way the problems are studied'. Many in the mathematics education community agree with Gardiner that much teaching overemphasizes the content elements of the subject to the detriment of developing mathematical processes. They argue that it is more important to learn, for example, the skill of working systematically than the meaning of rotational symmetry. One is a mathematical fact; the other is a process that is useful in doing a range of mathematics.

Why should mathematics be taught?

All teachers are under pressure to produce good examination results, which can cause some to feel that a good set of results is the main purpose of their teaching. Although clearly this aspect of the job is high profile, it is important for you to consider the broader picture and examine why mathematics holds its position as part of the core curriculum.

If mathematics is mainly a tool for solving problems, then its reason for being in the curriculum is clear; it is so that pupils can acquire the skills they need to solve problems. If, on the other hand, mathematics is a fascinating body of knowledge or a means for appreciating patterns, then the reason for teaching it must be that it forms part of culture, and that an understanding of mathematics is required before anyone can be considered fully educated. This is clearly a more difficult idea to articulate, but is nevertheless a perfectly reasonable justification for teaching the subject.

The Mathematical Association identify a series of mathematical goals that define what mathematics teachers are trying to achieve (Mathematical Association, 1995: 8):

The pupil should develop the ability to:

- read and understand a piece of mathematics
- communicate clearly and precisely using appropriate media
- work clearly and logically using appropriate language and notation
- use appropriate methods for manipulating numbers and symbols
- operate with shapes both in reality and in the imagination
- apply the sequence 'do, examine, predict, test, generalize, prove'
- construct and test mathematical models of real-life situations
- analyse problems and select appropriate techniques for their solution
- use mathematical skills in everyday life
- use mechanical, technological and intellectual tools efficiently.

This list seems to include all the expected references to utilitarian aims, and makes clear reference to mathematical communication, but could be seen to under-represent the purist perspective. 'Work clearly and logically' is a mathematical skill concerned equally with thinking and with solving problems, and the concepts of generalizing and proving are key ideas from pure mathematics, but there is only one mention of the word 'understanding', and no explicit reference to appreciation of, or interest in, the subject.

The two bullet points on communication and the mathematical skills in everyday life are clearly important. On a basic level, mathematical language is part of everyday communication, and includes the huge number of graphic presentations used in the media to convey information. Hence it is important to teach mathematics so that pupils become informed citizens, who are able to understand information presented to them in a variety of graphical forms.

Teaching mathematics is sometimes justified by the argument that it trains the mind (for example, Smith, 2004), and is thus an aid to learning in other disciplines. As a justification in itself, this seems to overstate the case, and we need evidence of exactly what this training of the mind really means. It is more commonly accepted that acquiring general thinking skills is a cross-curricular aim of education rather than a justification for teaching any one particular subject.

Finally, in this section, we should mention the more general humanistic justifications for teaching mathematics. A study of mathematics contributes to societal values, how people feel about themselves and their environment (Bishop, 1988). Mathematics can provide people with a feeling of control over their environment, and therefore it increases a sense of power through knowledge. We are able to control events because we feel that they are predictable. Second, the study of mathematics suggests that problems can be solved, if not in full then in part. Mathematics thus reinforces the view that advances in society are possible and that aspirations to a better way of life are realistic. Third, mathematics reinforces a belief in rationalism. Things can be explained through logical argument; we can convince others of the correctness of our thinking through reason. Put together, these three justifications mean that mathematics helps us to feel more comfortable about the world where we live.

POINT FOR REFLECTION

Examine your own background to learning mathematics. In a utilitarian age, it is easy to justify mathematics in terms of its usefulness, but as specialist mathematicians, we should consider that the subject stands strongly in its own right. How would you convince someone that school mathematics should be studied because it is a worthwhile area of study that is part of human culture?

Numeracy and mathematics

'Numeracy' is one of those words whose meaning seems to have changed in recent years. Formerly, it was used to represent that subset of mathematics involving numbers, particularly understanding what numbers mean, and being able to perform calculations.

It then started to be used as shorthand for basic numeracy, understood as the sort of 'everyday' mathematics that all school leavers would need to cope with. This is the sense in which the word is used in the Cockcroft Report. In Cockcroft (1982), numeracy is being comfortable working with numbers, but also it is the set of mathematical skills used in daily life. This definition is itself open to interpretation, but most would concede that essential mathematics includes aspects of interpreting data, or using graphs, maps and scales that are not predominantly number work. It is in this sense that politicians and the media often use the word.

When, in 1996, the government set up a review into the teaching of mathematics in primary schools, the title chosen was the National Numeracy Project. Numeracy's connection with number work is retained in the working definition given by Askew et al. (1997), that numeracy is the ability to 'process, communicate, and interpret numerical information in a variety of contexts', but as the National Numeracy Project came to an end, its name was partly retained in the title of *The National Numeracy Strategy: Framework for Teaching Mathematics* (DfEE, 1999b). Hence in primary schools, the word 'numeracy' became almost synonymous with the word 'mathematics'. Being numerate is understood as the ability to do mathematics (rather than any subset of mathematics).

There is some debate about whether there is still any useful distinction to be made in the use of the two words 'numeracy' and 'mathematics'. The discussion given by Tanner and Jones (2000) sees numeracy as a foundation for the whole of mathematics, rather than the whole thing. Numeracy involves 'an interaction between mathematical facts, mathematical processes, metacognitive self-knowledge, and affective aspects including self-confidence and the enjoyment *of number work*' (Tanner and Jones, 2000: 146, our italics). Our own view is that numeracy is virtually synonymous with mathematics, but with two differences. The first distinction is that numeracy is a slightly more active word than mathematics. Numeracy is less likely to be understood as a body of knowledge, and is more associated with doing mathematics. The second distinction is that numeracy has an (undefined) upper limit. Higher levels of study will always be called mathematics; lower levels of study may be called mathematics or numeracy.

The lack of a short verb to describe being numerate has caused difficulties for those wanting a headline or soundbite. In 1997, the Department for Education and Employment described the main functions of education as ensuring that every child can read, write and add up (DfEE, 1997). Mathematical basics have here been reduced to the very basic! In another example, when an employers' leader criticized the levels of school leavers' English and mathematics, one newspaper headline read, 'Bosses say school leavers can't read, write or count' (Stewart, 2005). The rest of the article makes no reference to the ability to count, but refers instead to mathematics and problem-solving skills. The work that goes on in mathematics classrooms is thoroughly trivialized by this soundbite usage, where an ability with basic mathematics is equated to the ability to add up or to count.

However, the most recent National Curriculum (DFE, 2014) makes no mention of numeracy. It is possible that the use of the word numeracy has had its day and will decline over the coming years.

Evidence from the research

The philosophy of mathematics is a rich area for discussion. We have discussed the nature of the subject in terms of the purist/utilitarian viewpoints already, but there is also considerable debate about the extent to which mathematics is a social activity.

Traditional philosophers of mathematics treat it as a subject that stands on its own. It needs no input from other disciplines; it remains constant over time, and it is not affected by social constructs in any way. 'A theorem is true regardless of whether it is proven by a human, a computer or an alien' (Tegmark, 2003: 13). Others (for example, Hersh, 1998) argue the contrary: that mathematics must be understood as a human activity that has evolved historically, and which takes place in a social and cultural context. Such writers contend that there is a human dimension to the way in which mathematicians work. In this sense, mathematics is invented, rather than discovered.

This is one of the key discussions of mathematical philosophy. One group believes that mathematical truth is certain, that it is incontestable and entirely objective. For example, Kassem (2001: 72) reports 'a deep seated notion that the subject is value-free, independent of society and an exemplification of absolute truth'. In a similar vein, Shapiro (2000: 257) states that mathematical statements are true or false 'independent of the language, mind and social conventions of the mathematician'. This is known as the truth view of mathematics. Holders of this view are accused of an idealized view of the subject, ignoring how mathematics is, in favour of how it ought to be (Körner, 1960). A commonly accepted view today is that mathematics is constructed; its truths are subject to argument, and may at any time in the future be challenged and revised. This is known as the constructivist view of mathematics.

All agree that mathematical truths are proved from axioms, using the rules of inference. The constructivists argue that there is a fundamental fallacy in regarding mathematics as absolute truth. All mathematics uses deductive proof to demonstrate truths based on axiomatic starting points. But whatever axioms are chosen, they are simply chosen and not absolute. Examples of axioms may be that $1 + 1 = 2$, or that the number of natural numbers is infinite. These may be thought to be above question, but their existence weakens the absolute truth view of mathematics. Ernest (1991: 13) argues that 'deductive logic only transmits truth, it does not inject it, and the conclusion of a logical proof is at best as certain as its weakest premise'.

Lakatos (1978) demonstrates the weakness of seeking certainty in mathematics. Any mathematical system depends on a set of assumptions. In order to prove an assumption we need to make earlier assumptions, and so on. We can never be free of the assumptions. The role of the mathematician is to reduce the number of assumptions to the smallest number possible.

> Mathematical truth ultimately depends on an irreducible set of assumptions, which are adopted without demonstration. But to qualify as true knowledge, the assumptions require a warrant for their assertion. But there is no valid warrant for mathematical knowledge other than demonstration and proof. Therefore the assumptions are beliefs, not knowledge and remain open to challenge, and thus to doubt. (Ernest, 1991: 14)

There is one further weakness in the truth view of mathematics: just as axioms are stated without proof, so the rules of deductive logic are themselves unprovable. Thus the foundations of mathematics as an unquestionable truth are weakened further.

So if we reject the philosophy of mathematics as truth, it becomes necessary to articulate a philosophy that mathematical truths are open to argument and can be refined over time. According to Hersh (1998), mathematical philosophy should not be about seeking universal truth. It should seek to give an account of mathematical knowledge as it really is: fallible, evolving and as subject to argument as every other branch of knowledge.

As discussed earlier in the chapter, mathematics is more than a body of knowledge; it is also an activity of gaining knowledge and understanding. As soon as we embrace this change in viewpoint, then mathematics becomes a human activity, and part of human civilization (Voskoglou, 2018). Older philosophers saw mathematics as separate from other fields of human learning, but once it is accepted that mathematics is not infallible, the subject becomes part of the broader human knowledge that includes the sciences. According to this perspective, mathematics is part of society and hence a product of the culture that produced it. The development of mathematics is then subject to societal influences; it has values and cultural influences. As Paul Ernest points out, the aims of teaching mathematics are 'expressions of values, and thus the educational and social values of society or some part of it are implicated in this enquiry' (Ernest, 2016: 2).

Putting a greater emphasis on the social side of mathematics has been propounded as a way of making the subject more interesting to more pupils. Lingard (2000) makes a strong case that learning about the history of mathematics may help motivation and hence achievement. The history of mathematics shows that the subject has developed over time (and is still developing) and reminds pupils that mathematicians are human.

If mathematics consists of a set of universal and incontrovertible truths, then it should not include apparent inconsistencies. At a simple level, it is possible to argue that the recurring decimal 0.99999... is equal to one, and also that it is slightly less than one (see, for example, the argument on the *Better Explained* website, in 'useful websites' on page 16). One famous challenge to the position of mathematics as a body of logical truths comes in the form of Russell's paradox. This suggests that we can separate out all sets into two piles, pile one for sets that are members of themselves and pile two for sets that are not members of themselves. If we then consider the set of all sets that are not members of themselves, then we have a paradox. We do not know which pile this set belongs in, because it appears to be a member of itself if and only if it is not a member of itself. The paradox illustrates the counterintuitive fact: it is possible to find illogicalities in mathematics!

POINT FOR REFLECTION

Consider the extent to which mathematics is influenced by society. If mathematics is constructed rather than an absolute body of truth, then the social context of the time should influence how mathematics develops, and the sort of mathematical dialogue that goes on. Would mathematics develop in a totalitarian society in the same way as in a liberal democracy? Consider examples of how the mathematics would be independent of context, and examples where the contrary is the case.

Further reading

Advisory Committee on Mathematics Education (ACME) (2011) *Mathematical Needs: The Mathematical Needs of Learners*. Available at www.acme-uk.org/media/7627/acme_theme_b_final.pdf

This well-researched publication was influential in how the National Curriculum has developed. It starts with an interesting review of the nature of mathematics and goes on to discuss key features that should be present in any mathematics curriculum. You should find the review useful as it draws on the 'big ideas' of teaching mathematics and, as such, is very thought-provoking. The review makes a number of recommendations that the authors believe should drive national policies on mathematics education.

Ernest, P. (2016) 'An overview of the philosophy of mathematics education', *Philosophy of Mathematics Education Journal*, 31 (November).

The author is an acknowledged expert in the field of philosophy in mathematics education, and edits a journal on the subject. In this thought-provoking article, he discusses what is meant by having a philosophy of mathematics education. Much of the article raises questions for consideration. He covers the big questions about the nature of mathematics, how it relates to society, and why we teach it, and also considers how a philosophy of mathematics education relates to broader philosophical perspectives.

Useful websites

The Teachers' Standards can be found on the website of the Department for Education (DfE), at www.gov.uk/government/publications/teachers-standards

The journal *Philosophy of Mathematics Education* can be found online at http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/

The *nrich* website, run from Cambridge University, has discussion, articles and enrichment problems that are linked in with the school curriculum. The home page is https://nrich.maths.org/

The *Better Explained* website, with a discussion of 0.9 recurring, is at https://betterexplained.com/articles/a-friendly-chat-about-whether-0-999-1/

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