Introduction to Block 1

Statistical ideas can be expressed using alternative and complementary forms: words and numbers, pictures and ICT. Block 1 of this book looks at statistical ideas from the point of view of how they can be expressed using words and numbers, while pictures and ICT are the central themes of Blocks 2 and 3.

These first four chapters cover, in turn, the following ways of thinking: describing, comparing, interrelating and dealing with uncertainty. A variety of statistical 'big ideas' are explored – for example, relative and absolute differences, the notion of variability and the distinction between a statistical and a cause-and-effect relationship.

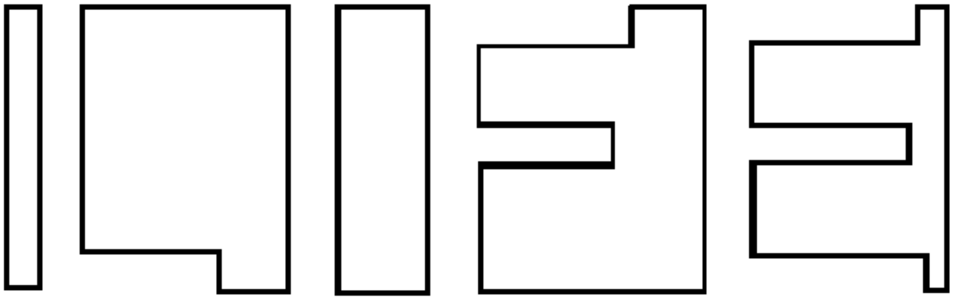
A number of useful and important teaching issues are offered; for example, the gambit of providing learners with 'telling tales' that are designed to illustrate an important statistical idea in an interesting and memorable context. Constructivism is an area of educational philosophy that describes how learners 'construct' meaning in their learning and you are asked to consider how this idea might relate to statistics learning. You will also be shown, and asked to use, a four-stage framework for tackling a statistical investigation.

1 Describing with Words and Numbers

In general, descriptions of things can be *verbal* or *numerical* in nature. This chapter explores some similarities and differences between these two forms of describing.

Task 1.0.1 Shapes

Spend about 20 seconds looking at the five shapes below. Then close the book and try to draw them as accurately as you can.



Some people have a good eye for shape and can do these sorts of tasks fairly easily. Others find it difficult to remember the details and perhaps tend to get mixed up, for example, when trying to remember shapes of similar type such as the last two. One problem is that there seems to be nothing that enables you to see these shapes as a whole – they appear to be just five unconnected things.

Task 1.0.2 The Bigger Picture

Place two straight edges horizontally across the top and bottom of the drawings above. Now look not at the shapes themselves but at the spaces in-between. Does a word jump out at you? Keep looking until it does. Eventually, you should find that that the individual shapes seem to have a 'life' of their own. Now have another go at drawing the five shapes. Is the task suddenly easier?

So, these five shapes are not unconnected after all. It is the experience of most people that, having grasped the 'bigger picture', the component parts are subsequently easier to discern. It is a facet of human nature that different people attend to different things and, indeed, the same person attends to different things at different times. This phenomenon is sometimes referred to as 'stressing and ignoring', a phrase that will be used more than once in this book. It describes an important aspect of learning, whereby, in order for a learner's attention to be directed to one aspect of a problem, it is helpful to shut out its other features.

It seems that information is hard to absorb when each fact stands in isolation from others, unconnected to a wider context or some overarching 'big idea'. Knowing about these big ideas will help provide learners with coherence and meaning to the many facts and techniques they are expected to acquire from a statistics course. An awareness of the big ideas enables a newly learned technique to be applied to other situations in the future, not just the one in hand. As novelist George Eliot observed:

> It has been well said that the highest aim in education is analogous to the highest aim in mathematics, namely, not to obtain *results* but *powers*, not particular solutions, but the means by which endless solutions may be wrought.
>
> (Quoted in Carlyle, 1855; italics in original)

Task 1.0.3 The Big Ideas of Statistics **C**

Spend a few minutes looking at Table 1.1, which lists some of the know-how and techniques of statistics (row 1) and some big ideas (row 2). To help you to get a better feeling for the nature of a big idea, try to match up each item in row 1 with one or more corresponding terms in row 2. Are there any terms here you are unfamiliar with? How might you find out what they refer to?

TABLE 1.1 Matching Up

Know-how and techniques	Understanding and knowing how to calculate or draw: the range, the median, a bar chart, a scatter plot, the quartiles
Big ideas	Summarising, measures of location, spread and variation, interrelating

When you see a 'C' in the task header, remember to check 'Comments on Tasks' at the end of the book.

In common with many other disciplines, one big idea stands out from all others and that is *describing*. As you can see from the main title of this chapter, this is its central theme. Section 1.1 suggests that, in statistics, the central building block of thinking statistically is data. As Sherlock Holmes once remarked to Dr Watson, 'You were trying before you had sufficient data, my friend; one can't make bricks without straw' (quoted in Hanson, 2001, p. 628).

The two main themes in the first section are sampling and designing a questionnaire. Section 1.2 then explores questions of measurement, and you will be introduced to a number of different types of measuring scales. Data can involve both very large and very small numbers and how these are handled and represented is explored in Section 1.3. Summarising data often takes the form of calculating averages and spreads: these ideas are considered in Section 1.4. The final section looks at teaching and learning issues.

1.1 DESCRIBING WITH DATA

Depending on each individual's personality and training, everyone sees the world in a slightly different way. An artist's ways of seeing are predominantly tactile and visual, exploiting colour, texture and shape. A historian draws on and produces descriptions and analyses of past events, in order to gain insights about human behaviour. An author will describe her or his world with words, using metaphor, onomatopoeia and, on occasion, poetry. Each of these ways of seeing is valuable, and, when taken together, provide rich, diverse and complementary perspectives on human concerns.

What, then, is the particular *statistical* view of the world? It is essentially one based on quantitative descriptions, where the world is viewed on the basis of measures and counts, otherwise known as statistical information or data. Outside a narrow educational context, data are rarely collected without a clear reason or purpose. Typical questions that may inspire the need for data collection may be of the form, 'How big is A?' or 'Is A bigger than B (and by how much)?' or 'How is X related to Y?'. The nature of these questions is considered in more detail in, respectively, the first, second and third chapters of each of the first three blocks.

Once data have been collected, they need to be analysed in order for interesting or unexpected patterns to be identified. These patterns must then be interpreted in the context of the original question that started the investigation.

Four stages have now been identified that correspond to the following four important phases of a statistician's work. In summary, these are:

P stage: pose a question;
C stage: collect relevant data;
A stage: analyse the data;
I stage: interpret the results in the context of the question.

These four stages are referred to as the *PCAI* framework for conducting a statistical investigation: they are considered more fully later in the book, particularly in Chapters 2 and 14.

In this section, you will focus on the *C* stage of data collection, looking at two important aspects of how data are collected. The first concerns exploring how and why samples are taken in statistical work. In the second, you will look at how social data are collected, raising issues of polling and questionnaire design.

Sampling

A *census* refers to data collection where all the people or items of interest are measured or surveyed. However, when collecting information about a large population, it is often too inconvenient or expensive to measure every item: you must make do with taking a representative sample. This might take the form of choosing items randomly from a production line and measuring them to ensure that the values fall within acceptable tolerance limits. Alternatively, you may wish to survey the opinions of customers on how satisfied they are with the layout of a store or invite people to reveal their voting intentions just before a general election.

Task 1.1.1 Thinking it Through **C**

Imagine that you wish your learners to carry out a poll of television viewing habits of young people. This will involve taking a sample from the learners at their school. Before carrying out the poll, there are certain questions that they will need to think through about choosing their sample. Write down two or three of these questions. How might you help learners to tackle these questions sensibly?

Inevitably, in any discussion with learners about choosing a sample, a key point to emerge will be the importance of choosing a *representative* sample. (Sampling is looked at again in Section 14.2.)

Questionnaire Design

Although many scientific and industrial data are collected by direct measurement, 'social' data (for example, people's opinions on social issues like crime and health) usually require the use of a well-designed questionnaire. Unfortunately, the wording of questions in newspaper questionnaires is sometimes blatantly loaded or biased. Look at this one published several years ago in the *Daily Star* under the headline 'We've had ENOUGH!'

> 1. I believe that capital punishment should be brought back for the following categories of murder:
>
> Children [ ] Police [ ] Terrorism [ ] All murders [ ].
>
> 2. Life sentences for serious crimes like murder and rape should carry a minimum term of:
>
> 20 years [ ] 25 years [ ].
>
> 3. The prosecution should have the right of appeal against sentences they consider to be too lenient [ ].
>
> Tick the boxes of those statements you agree with and post the coupon to: VIOLENT BRITAIN, Daily Star, 33 St. Bride St., London EC4A 4AY.

The fact that this article appeared alongside articles with headlines such as 'My mother's killer runs free' and 'Hang the gunmen' is likely to have biased the response to these questions. But it is the nature and limited range of choices offered that distorts the survey even more. For example, there are no tick boxes available for people who do not support the death penalty or who favour alternative forms of punishment to imposing longer jail sentences.

It may not be too surprising to learn that, of the 40 000 readers who responded (and just how representative were *they* of the readers of the *Daily Star* or the population as a whole?), 86% favoured restoring the death penalty for murder, 92% wanted a 25-year minimum sentence for serious crimes of violence and 96% supported the prosecution right of appeal against 'too lenient' sentences.

In this *Daily Star* example, the bias was obvious. However, bias can occur, even in a well-designed questionnaire where there is no intention to deceive. As Steven Barnett (former chairman of the Social Research Association) argued in an article in the *Guardian* newspaper in 1989:

> Ultimately, however reliable the sample, social surveys consist of an aggregation of very short dialogues between two complete strangers. When these dialogues attempt to address intimate questions of feelings and opinions on social issues, the scope for misinterpretation is considerable. (p. 5)

As can be seen from the *Daily Star* questionnaire, how the question is worded can have a crucial impact on the response given. (For example, many people are strongly in favour of the democratic right to withhold their labour in an industrial dispute, but would personally draw the line at striking.)

Task 1.1.2 Your Opinion about Seekers of Opinion

What are your attitudes to social research? Do you feel it provides worthwhile data or is largely a waste of time? If you do have negative feelings, try to note down precisely why you hold this view.

Some people think this sort of research is invalid on the grounds that, 'they didn't ask *my* opinion'. This is an unfair criticism – provided the sample has been sensibly chosen to represent, fairly, the overall population, its findings should provide a useful snapshot of opinion. Second, they may feel it to be deceitful, because sometimes surveys are used as a cheap device for pitching a sale about a related product. This is a valid point and a good reason for avoiding co-operation with certain 'surveys' conducted in the street or on the phone. Finally, people may think that surveys are a waste of time because the questionnaires they see are designed by school pupils carrying out a shopping survey or those in newspapers like the *Daily Star* example. This problem is compounded by the fact that many newspaper polls (for example, those soliciting voting intentions before an election) are carried out in a matter of days, with little time to check wordings or test accuracy.

Nevertheless, despite these obvious dangers and drawbacks, social research can fulfil an important and valuable function in identifying areas of need in society and trying to ensure that resources are used sensibly. This sort of justification for social research is rarely explained to the general public and is certainly an aspect of statistics that should be discussed with learners. As Steven Barnett (1989) remarked:

> Whatever the hazards, however, survey research has a critical role in the development of much social policy. Patterns of employment, of health care, of transport, or of criminal activity pose awkward social questions requiring urgent solutions which compete for scarce resources. Results of large-scale social research can therefore provide the basis for decisions which affect people's lives. For this reason, the responsible social scientist will take every conceivable measure to ensure that the scope for misunderstandings are reduced to an absolute minimum. (p. 5)

Task 1.1.3 Eliminating Questionnaire Bias **C**

How do you think reputable polling agencies like MORI and Gallup try to ensure that the questions they ask are as clear and free from bias as possible?

1.2 DESCRIBING BY MEASUREMENT

The philosopher Aristotle (384–322 BCE) described a human being as 'a rational animal', believing in the power of reason as a natural human state. This view largely disappeared in the 'dark ages' (roughly 400–1300) in favour of the view that people should act on the basis of faith and emotion. René Descartes (1596–1650) was a key supporter of the re-emergence of rationality and argued that people should act according to the evidence of their senses and be informed by the power of reasoning rather than making decisions on the basis of divine inspiration.

However, rational decision-making is still treated with suspicion in certain quarters – based on a fear, perhaps, that individualism is being suppressed by the imposition of conformity, with no room to express one's individual personality. This is an attitude of mind raised in some of the quotations listed in Task 0.1. People are correct to be sceptical when attempts to achieve rationality are conducted in a crass or inappropriate manner. People fear that decisions based on quantification and measurement may miss the point – perhaps this runs the risk of including only the easy-to-identify factors and under-representing more subtle human characteristics. As French philosopher and mathematician Blaise Pascal (1623–62) expressed it:

> The heart has its reasons, which reason does not know. (1670, IV: 277)

Perhaps the moral here is that there are many ways of seeing the world, of which a useful and sometimes illuminating means involves quantification and statistical analysis.

Measuring Scales

The purpose of measuring is to describe things and the two most basic forms of description are words and numbers. In the next task, you are asked to think about these two ways of describing.

Task 1.2.1 Words or Numbers **C**

What are some strengths and weaknesses of using 'words' and 'numbers' to describe things? List some examples of things that are better described by words and others that are better described by numbers.

When different but related words are used as descriptors, it is often helpful to be aware how they relate to each other. Sometimes it is possible to elicit a natural ordering for these words that may not have been obvious initially.

Task 1.2.2 Only Words? **C**

Look at the three lists of words below. Can you think of any sensible way of ordering them?

(a) sit, walk, roll, crawl, run;

(b) gold, wood, silver, ruby, paper;

(c) rayon, cotton, linen, silk, wool.

Task 1.2.2 shows that, depending on context, certain sets of words do *sometimes* have a natural order. Clearly, there are other sets of words that *always* possess a natural ordering (for example, small/medium/large, months of the year, days of the week, and so on). However, care must be taken when the words are used in a cyclic arrangement – to take the example of days of the week, does Monday come before or after Thursday? The answer is that it depends which Monday and which Thursday you are referring to.

Turn your attention now to *numbers* and their properties for describing things. As you will see from the next task, numbers also operate in different ways, depending on context.

Task 1.2.3 Twice **C**

Read the statements below. If you feel that any are incorrect, write down why.

(a) Manchester United scored six – twice as many as their rivals Charlton Athletic.

(b) She turned up at 6 p.m. – twice as late as the advertised starting time of 3 p.m.

(c) He turned up six hours late – twice as late as his sister who came only three hours after the advertised starting time.

(d) The temperature was 15 °C, but the next day it was twice as hot, at 30 °C.

(e) The first tremor measured 2.0 on the Richter scale. That was followed by one twice as strong – it measured just over 4 on the Richter scale.

The next two sub-sections look at two key ideas in the area of measurement – the Stevens taxonomy for sorting out measures and their properties, and the distinction between discrete and continuous scales of measure.

The Stevens Taxonomy

There has long been confusion and debate about measurement and the nature of measurement scales. This became a major issue in the field of psychophysics in the 1930s. (Psychophysics is concerned with describing how an organism uses its sensory systems to detect events in its environment.) In 1932, a British committee investigated questions such as: 'By how much must the frequency of a sound be raised or lowered before a person can *just* detect a difference in pitch?' (Such a difference is referred to as a *limen*.) The committee concluded that subjective judgements of this sort could not be a basis of measurement.

S.S. Stevens (1906–73)

This is where Stanley Smith Stevens, a professor of psychophysics at Harvard University, joined the debate. He argued that the fault lay not with the particular measures, such as the limen, but with the fact that there existed no clear schema for understanding the nature of measurement. In a seminal paper (Stevens, 1946), he described a taxonomy of measurement based on four classes of scales. He named these 'nominal', 'ordinal', 'interval' and 'ratio' scales and they are discussed below.

A *nominal* (or *naming*) *scale* of measurement is used for named categories such as race, national origin, gender, surname, and so on. For the purposes of counting such data, it is common to code the names with numbers: for example, 0 = male and 1 = female. Sometimes numbered data are actually nominal, in the sense that the number is no more than a label, with no conventional quantitative significance: for example, the numbers on the shirts of footballers or marathon runners, telephone numbers and university learner numbers are all nominal. All that can be done with nominal measures is to count how many fall into each category (and maybe compute the corresponding percentages).

An *ordinal* (or *ordered*) *scale* of measurement involves data which can be ranked in order (first, second, third, and so on), but for which the numbers cannot be used for further calculation. For example, a questionnaire on attitudes to university fees may ask respondents to indicate their attitude on a five-point scale, ranging from 1 = very strongly opposed to 5 = very strongly in favour. As with nominal data, the number of responses falling into each category can be counted and percentages calculated, but now the values can be ordered by size. (Ideas of ordering were raised in Task 1.2.2.)

An *interval scale* of measurement is a rather subtle concept – numbers are used for measurement of the amount of something, but the scale is such that the zero is arbitrary. This idea occurred in Task 1.2.3, with the questions about time of day and temperature. Zero degrees (0 °C) does not mean 'no temperature'. It is an arbitrary point on the temperature scale associated with the temperature at which water freezes at sea level (under normal atmospheric pressure). Contrast this with measures such as mass or length: a length of 0 cm does mean 'no length' and something that is 20 cm is twice as long as something that is 10 cm. Most psychological tests, such as measures of personality and academic 'ability', are interval measures. (If a learner receives zero on an exam, it does not mean that he or she knows nothing about the course.)

Interval data have all the properties of nominal and ordinal data but, additionally, the intervals between adjacent values are equal. This means that they can be added and subtracted (so, 40 °C is 10 °C warmer than 30 °C or a person may increase their IQ score by 5 points). However, the operations of multiplication and division do not apply appropriately to interval scales. For example, as you saw in Task 1.2.3, a temperature of 30 °C is not twice as warm as a temperature of 15 °C.

A *ratio scale* of measurement has all the properties of an interval scale but, additionally, the operations of multiplication and division can be used meaningfully. Thus, it is fair to say that an age of 16 years is twice as much as an age of 8 years, or that someone weighing 66 kg is twice as heavy as someone weighing 33 kg. Change the years to days or the kilos to pounds and the relationship of 'twice as much' is unaffected. Examples of ratio measurements include mass, length, speed or acceleration and, unlike with interval scales, zero really does mean 'none' of whatever quantity is being measured.

You can consolidate your understanding of Stevens's taxonomy of measuring scales by tackling Task 1.2.4.

Task 1.2.4 Testing Your Take on the Taxonomy **C**

(a) Classify the following measures according to Stevens's taxonomy:
- (i) A questionnaire asking: 'How would you rate the service of our catering staff?'
  Excellent ( ) Good ( ) Fair ( ) Poor ( )
- (ii) Windspeed, measured in knots.
- (iii) Bathroom scales, measuring in kg.
- (iv) The UK is made up of four separate countries: England, Scotland, Wales and Northern Ireland.
- (v) Wind force, measured on the Beaufort scale (0 = calm, 1 = light air, 2 = light breeze, …, 12 = hurricane).

(b) Is it possible for temperature to be measured on a ratio scale?

(c) How can time of day and time elapsed be distinguished using the Stevens taxonomy?

The Discrete/Continuous Distinction

There is another means of distinguishing types of measurement scale, namely, the difference between discrete and continuous scales.

Task 1.2.5 Grammar, Timothy! **C**

(a) Overheard: Timothy, aged 9, was in a bookshop with his mum. Holding up the new Harry Potter book, he said, 'Mum, guess how much pages are in this book'.

Why is Timothy's grammar incorrect?

(b) Distinguish the meanings of the words 'fewer' and 'less'.

(c) What 'big idea' connects your answers to parts (a) and (b) and how does it relate to statistics?

The distinctions in parts (a) and (b) above are essentially the same. Words like 'how many' and 'fewer' refer to measures of discrete, separate, countable items, whereas 'how much' and 'less' refer to something that cannot be counted out, such as amount of water, size of slice of a pie, and so on. The terms used in statistics to make this distinction are 'discrete' and 'continuous', respectively.

The key feature of a continuous variable is that, as the name implies, one value flows continuously into the next with no gaps or steps. Between any two values of a continuous variable there is an infinite number of other possible values. Examples of continuous variables include height of plants in a nursery, air temperature and waiting time in a dental surgery.

Contrast this with a discrete variable, where the possible values that it can take are restricted, with gaps between adjacent values. Examples include size of household, marks scored in a test and outcome when rolling a die. It is not possible to have a household size or die score of, say, 3.17. (It is, of course, possible to have an *average* household size of 2.4 or an *average* die score of 3.17, but these numbers arise as a result of calculating an average and such averages are not usually an actual item of the data from which they were calculated.)

Task 1.2.6 Distinguishing Discrete and Continuous Measures **C**

Look at the measures below and classify them into 'discrete' or 'continuous' measurements.

(a) A child's foot length.
(b) A child's shoe size.
(c) The duration of a film.
(d) The time taken to run a race.
(e) The number of runners in a race.
(f) Air temperature.
(g) The number of matches in a box.
(h) The speed of a car.
(i) The number of goals scored in a season.
(j) Time displayed on a dial watch.
(k) Time displayed on a digital watch.
(l) Annual salaries of teachers.

Now look at the two sentences below and then tackle Task 1.2.7.

The UK is made up of four separate countries: England, Scotland, Wales and Northern Ireland.
The UK is made up of four separate countries: Wales, England, Scotland and Northern Ireland.

Task 1.2.7 Wales Has Been Moved! **C**

The two sentences are identical except that, in the second, the word 'Wales' has been moved to a different position in the sentence. Clearly, these sentences have exactly the same number of characters, yet they do not occupy the same number of lines of text – a matter of great importance to, say, a newspaper editor.

Try to explain this phenomenon, using the ideas of this section.

One difficulty with making the discrete/continuous distinction is that, in the real world, measurement of a continuous variable can never be carried out with perfect accuracy. Inevitably, rounding reduces the number of possible values that the variable can take from a theoretical infinity of possibilities to something finite. This means that, in practice, *all* measurement can be thought of as being discrete. However, the distinction is still a useful one to make, particularly when it comes to displaying data graphically.

1.3 LARGE AND SMALL NUMBERS

There is no doubt that most measurement involves numbers; sometimes these can be huge numbers and at other times extremely tiny. It is often hard to know how to react to being told certain numerical information without knowing whether it is a lot or a little. A 'mental muscle' that needs regular exercise is the ability to make sensible, 'ball-park' estimates, particularly with large and small numbers. Estimation tasks are a useful and entertaining way of developing your number feel – finding out how big things are to some order of magnitude.

Rough Calculations

Some facts you either tend to know or not, as the case may be. (How far is it from John o'Groats to Land's End or what is the approximate population of the world?) Other 'facts' may not be known precisely, but with a bit of intelligent guesswork and a few 'back-of-an-envelope' calculations, a rough estimate can be made. Sometimes, for certain purposes, a rough estimate is all that is required. For example, in 2004, a van driver drove through Dover with 1 million cigarettes in the back. When questioned, he claimed these were for his 'personal use'. On the basis of a rough calculation, Customs officials estimated that even a heavy smoker would have enough smoking material for 'personal use' to last for 50 years!

Task 1.3.1 Rough Estimates **C**

Make a rough guess and then an estimate of the following. As you do so, think about the estimates and calculations that you carried out along the way and what 'facts' and assumptions they were based on.

(a) How long will it take for 1 million seconds to tick by? (Make a guess before getting out your envelope.)

(b) How long would it take for 1 billion seconds to tick by? Note that modern usage of the word 'billion' refers to 1 000 million. (Make a guess before making your estimate.)

(c) The population of the world is roughly 6.5 billion people. How many people in the world die each day?

(d) The population of the UK is roughly 57 million. How many cigarettes are smoked annually in the UK?

(e) How fast does human hair grow, in km per hour?

Having an understanding of the orders of magnitude of numbers is an important life skill that is best developed by letting learners explore a wide range of real-world contexts, which is something for which statistics lessons provide many opportunities. In the next task, you are asked to think further about how this skill can be developed.

Task 1.3.2 Adapting Tasks **C**

Take one of the five statements listed in Task 1.3.1 and adapt or extend it for use with another learner.

If you used a calculator with Tasks 1.3.1 and 1.3.2, you will, no doubt, have come into contact with numbers expressed in scientific notation (also sometimes referred to as 'standard form'). This notation is inescapable when calculators and spreadsheets are used and is a topic that learners from age 14 on need to be able to handle with confidence. It is considered further in Task 1.3.4.

Visualising

In 1999, the eyes of the sporting world were focused on Wales with the opening of the Millennium Stadium, which was built on the historic Cardiff Arms Park site. Designed for all-year, all-weather use, the stadium was the first in the UK with a retractable roof and the first of its type in the world with 75 000 seats. If you have never attended a major public event, it is hard to imagine what 75 000 people gathered together in one place looks like, let alone what it sounds like when they all start shouting or singing at the same time. What would help you or a learner to get a feel for a number as large as this?

Writer Bill Bryson (2004) has a knack of making statistical information both meaningful and palatable to his readers, by providing a helpful mental picture of the magnitudes involved. Here are a few examples taken from his book, *A Short History of Nearly Everything*.

- A typical atom has a diameter of 0.00000008 cm. … Half a million of them lined up shoulder to shoulder could hide behind a human hair. (p. 176)
- An atom is to a millimetre as the thickness of a sheet of paper is to the height of the Empire State Building. (p. 177)
- Avogadro's number, the number of molecules found in 2.016 grams of hydrogen gas (or an equal volume of any other gas at the same temperature and pressure), is equal to 6.0221367 × 10^23, which is equivalent to the number of popcorn kernels needed to cover the US to a depth of 9 miles, or cupfuls of water in the Pacific Ocean, or soft-drinks cans that would, evenly stacked, cover the Earth to a depth of two hundred miles. (p. 139)
- Astronomers today believe that there are perhaps 140 billion galaxies in the visible universe. … If galaxies were frozen peas, it would be enough to fill a large auditorium – … say … the Royal Albert Hall. (p. 169)
- The most striking thing about our atmosphere is that there isn't very much of it. It extends upwards for about 190 kilometres … if you shrank the Earth to the size of a standard desktop globe, it would only be about the thickness of a couple of coats of varnish. (p. 313)
- [On how far back the Cambrian era is.] If you could fly backwards into the past at a rate of one year per second, it would take you about half an hour to reach the time of Christ, and a little over three weeks to get back to the beginnings of human life. But it would take you twenty years to reach the dawn of the Cambrian period. (p. 395)

Task 1.3.3 Visualising Facts **C**

Here are some facts. How would you try to visualise these numbers?

The capacity of the Millennium stadium, Cardiff: 75 000 people.

The capacity of the Centre Court at Wimbledon: 15 000 people.

The capacity of the London Palladium theatre: about 2 200 people.

The length of time humans have been around (4 million years), compared with the length of time the earth has been around (4.6 billion years).

Here are some of the examples given in this section of very large and very small numbers.

- The number of seconds that have elapsed since the creation of the earth is about 15 000 000 000 000 000 000.
- There are perhaps 140 billion galaxies in the visible universe.
- Human hair grows at roughly 0.000000014 km per hour.
- A typical atom has a diameter of 0.00000008 cm.

These are very extreme numbers and it is almost impossible to understand how large or how small they are. The problem lies with the sea of zeros that such numbers contain when expressed in conventional notation. Scientific notation (sometimes called standard form) provides an alternative way of representing numbers that overcomes this problem.

The number 37 200 can be written in standard form as 3.72×10^4 or sometimes as 3.72E4.
The number 0.00000437 can be written in standard form as 4.37×10^{-6} or sometimes as 4.37E⁻6.

Task 1.3.4 Converting Numbers into Standard Form **C**

(a) Using the two examples above as a guide, convert the following numbers into standard form.

15 000 000 000 000 000 000

140 billion

0.000000014

0.00000008

(b) Type these four numbers into a scientific or graphics calculator and press '=' (or ENTER) to check that your answers to part (a) are correct.

(c) What are the advantages of displaying these numbers in scientific notation?

Handling numbers in standard form needs a fair degree of practice, as you will see from Task 1.3.5.

Task 1.3.5 Calculating with Standard Form **C**

Predict what answer would be displayed on a calculator or spreadsheet to the following calculation: 2.4E36 – 2.4E12. Then perform the calculation and explain why this answer has resulted. What issues are there here that you might wish to share with learners?

1.4 SUMMARISING – AVERAGES AND SPREADS

So far in this chapter, descriptions have come in the form of individual words or numbers. An important feature of statistics is that, usually, *many* measures are made in order to describe things. This may take the form of an experiment or a survey in which a batch of data items together represents the description in question. (A *batch* is another name for a data set.) This move from single numbers to many numbers is a very big idea in statistics, one that is not always obvious to learners when they first study the subject.

Once data have been collected, whether from a survey or scientific experiment or something else, it is often hard to 'see the wood for the trees'. Some sort of organisation or simplification is necessary in order to gain a sense of what the data seem to be 'saying'. Where possible, data can usefully be organised into a table (table layout is discussed in Section 3.2) or summarised into a single figure (sometimes called 'a statistic').

If that single figure is a summary of where the data are *centred* (often called a measure of *location*), then the numerical summary is known as an *average*. If the aim of the summary is to understand how widely *dispersed* the data are, then a measure of *spread* is needed. Whatever the purpose of the summary (to describe the location or the dispersion of the data), they have one thing in common – summarising enables a large number of figures to be reduced to a single representative figure. This has obvious benefits in terms of providing a useful overview. However, summaries inevitably bring corresponding costs in terms of the loss of data and an investigator needs to steer a sensible path between these two aspects, namely, too much and not enough information.

The three best-known measures of location are the three averages, the mean, the mode and the median.

Task 1.4.1 Learner Definitions **C**

Here are plausible definitions of these three averages provided by a learner. Try to decide which is which and then, if required, improve the learner's definitions.

Definition 1: If there are 20 values, it's 10.

Definition 2: Add them together and divide by how many there are.

Definition 3: It's the biggest frequency.

At a more advanced level, two other useful 'averages' in some circumstances are the *geometric mean* and the *harmonic mean*.

The geometric mean: just as the arithmetic mean of two numbers is found by adding them and dividing by two, the geometric mean is found by multiplying them together and then taking the square root. The geometric mean of n numbers is found by multiplying them all together and then taking the nth root. (Finally, note that the geometric mean cannot be used when any of the values are ≤ 0.)

An example of where the geometric mean is useful is in calculations of vibrating frequencies in the tuning of musical instruments. The vibrating frequency of middle C on a piano is 256 Hz. The vibrating frequency of the C above middle C (one octave above) is twice this value, at 512 Hz. There are 12 semitone intervals between these two notes and in order to ensure that the intervals are equal, the geometric mean must be calculated. One way of explaining this is that, as the 12 semitone steps of the octave together multiply to 2 (in order to double the frequency from 256 to 512), each step is the twelfth root of 2, or roughly 1.059. So the semitone above middle C (named C sharp, written C#) has a vibrating frequency of 256×1.059, or roughly 271 Hz; the next semitone in the sequence (named D) has a vibrating frequency of $256 \times (1.059)^2$, or roughly 287 Hz; and so on all the way up to C above middle C, with a frequency of 512 Hz.

The harmonic mean: a common problem for learners is in the calculation of average speeds. If the average speed for the outward journey is, say, 40 kph and the average speed of the return journey is 60 kph, it is not the case that the average speed of the round trip is 50 kph. The arithmetic mean simply does not apply to this situation and what is required is the harmonic mean. This involves taking the following steps:

- take the reciprocals of the values 40 and 60: 1/40 and 1/60;
- find the arithmetic mean of the reciprocals: $(1/40 + 1/60)/2 = 5/240$;
- take the reciprocal of the result: the reciprocal of $5/240 = 240/5 = 48$.

So, the overall average speed of the round trip is 48 kph.

So, why does it work? In this particular example, further investigation by learners may reveal that the key to understanding calculations involving speeds is to concentrate on the time elapsed for each part of the journey. Since speed is equal to distance divided by time, there is an inverse relationship between speed and time. It follows that in order to average the times of the two parts of the journey, you need to find the average of the reciprocals of the two speeds.

A message here for learners is that it is always useful to consider problems from basic principles rather than mindlessly applying a formula.

Task 1.4.2 Create Your Own Task **C**

Now create a task of your own for helping learners to think about a measure of spread – the range. What might another measure of spread be?

In Section 1.2, you read about the Stevens taxonomy for classifying scales of measurement. This way of thinking is very helpful when checking which numerical summaries can and cannot be applied to certain measures. For example, since calculation of the mean involves adding numbers together, this choice of summary would not be suitable for nominal or ordinal data, where there is no uniform interval between values. This is easiest understood with specific examples, as the next task will reveal.

Task 1.4.3 When Averaging is Appropriate **C**

Note down four data sets, one each based on nominal, ordinal, interval and ratio measuring scales, respectively. Decide whether each can be summarised using the mean, the mode and the median. Try to make a general conclusion (in the form of a table) about which forms of average operate correctly with each of the four Stevens scales of measure.

1.5 PEDAGOGY: PREPARING TO TEACH A TOPIC

This section highlights some thoughts about teaching and learning statistics in relation to the topics of this chapter.

Task 1.5.1 Preparing to Teach

Choose a particular statistical topic that you may be required to teach a learner or group of learners (for example, types of average, ways of plotting statistical data or a topic in probability).

How would you prepare the lesson? Write down up to six key teaching issues that you would need to consider in the course of your preparation. Note that this is not an invitation to divide up the statistical topic into six sub-topics. Rather, you are asked to consider what aspects of learner learning you will need to bear in mind.

There is clearly no single correct answer to these questions of preparation of a topic – people prepare in various ways, depending on their own subject knowledge, particular interests and their perceptions of the needs of the learner(s) that they have in mind.

In this chapter, and indeed in all the chapters of the book as far as Chapter 12, the final section looks at pedagogic (that is, teaching) issues in terms of the following six themes. They will be referred to as the 'Preparing to Teach' framework (or PTT for short).

- *Language patterns* – learners may be using some of the technical terms and expressions already but perhaps without the standard mathematical meaning.

- *Imagery* that will help learners to create a richer inner sense of the topic, from which further connections can be made.
- *Different contexts* that can be used to enhance understanding and motivate learners by demonstrating that the topic has useful currency outside the classroom.
- *Root questions* – the sort of questions that prompted people to develop general techniques, thus giving rise to the topic.
- *Standard misconceptions* or different or incomplete conceptions.
- *Techniques and methods* that learners need to be able to master and recognise when to use them appropriately.

Task 1.5.2 Using PTT **C**

How do the six pedagogic themes relate to the topic that you chose in Task 1.5.1? Write notes under each of the six PTT headings.