# Investigations

Estimation, Large Numbers, and Numeration



# Investigations

# Estimation, Large Numbers, and Numeration

Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.

—Carl Sandburg, "Arithmetic"

athematics can be a powerful tool for students—and us—to make sense of the world around them. Without good estimation skills, we cannot approximate distances, or the number of people in a crowd, or how much our grocery bill will be, looking at a shopping cart full of groceries. Without an adequate understanding of large numbers, we cannot conceive of the enormity of a deficit of \$5,000,000,000,000 (five trillion dollars) or the need to change an area code to provide the telephone company with additional telephone numbers. Without a secure understanding of relationships between numbers, we are unable to fully comprehend fractions, decimals, and percentages.

In the middle grades, students "should understand numbers, ways of representing numbers, relationships among numbers, and number systems" (National Council of Teachers of Mathematics [NCTM], 2000, p. 214). In addition, they need to "understand meanings of operations and how they relate to one another . . . and compute fluently and make reasonable estimates" (p. 214). The activities in this chapter will encourage students to do the following:

- Estimate when working with large numbers and distances in real-life problems
- Problem solve strategies to find reasonable answers to motivating problems
- Investigate fractions and decimals in a variety of real-world and problem-solving situations
- Use hands-on activities to make connections between abstract concepts and the concrete models that represent them

What better way to develop good number concepts than through interesting and motivating activities that keep students *actively learning* mathematics!

#### \$1,000,000 LONG

This activity is an open-ended problem that encourages students to explore strategies, work collaboratively, make mathematical connections to social studies, and examine alternative solutions when dealing with large numbers. This activity asks students to use their knowledge of the length of one dollar bill to extrapolate the length of one million of them—the perfect opportunity for an authentic discussion of just how precise is precise?"

# HOW MANY STRIDES TO WALK AROUND THE EARTH?

This is another open-ended problem that requires each group to develop its own unique problem-solving strategies when pacing off a very large number of steps. The initial problem necessitates figuring out what a group's *normal* stride is. In the process of solving this problem, students work collaboratively, make mathematical connections to science, and use rounded numbers as they ponder precision and accuracy.

#### **RECTANGLES AND FACTORS**

Students use square tiles to discover the relationship between rectangular arrays and prime and composite numbers. When students use manipulative materials and reasoning skills to discover an abstract relationship, the learning is more meaningful and permanent. By associating numbers with their factors in an area model, students acquire a deeper and longer-lasting understanding.

#### VENN DIAGRAMS: LCM AND GCF

This activity helps students see the mathematical relationship between the greatest common factor (GCF) and least common multiple (LCM). The visual model supplied by the Venn diagram helps students associate abstract number theory with its visual representation and brings it into the "mind's eye" of the student.

#### **DESSERT FOR A CROWD**

This activity uses an actual recipe for a devil's food cake with marshmallow frosting. The original recipe serves eight people. Students, working together, change the recipe to bake enough cakes to feed their math class (or perhaps all of the students in the school).

#### $5 \times 5$ PUZZLE CENTS

This is an activity that makes practice with addition of decimals entertaining. Students are able to manipulate "coins" by cutting out the squares and moving them around a grid. The coins become a readily available manipulative that encourages the development of problem-solving strategies.

#### **CHOCOLATE CHIP COOKIES**

In this activity, the cost of each ingredient is listed, and students are asked to find the cost of each cookie, how much profit could be made if they were sold, and what percentage the profit represents. The next best thing to eating these cookies is thinking about eating them!

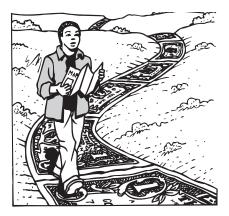
#### **MUSIC AND FRACTIONS**

Students use their fractions skills in this real-world activity that makes connections between music and mathematics. By adding and subtracting notes, students employ a mathematical skill and experience a real-world application for using fractions.

#### **MATHEMATICAL PALINDROMES**

Palindromes have played a fascinating role in both language and mathematics. In addition to palindromic words and phrases, there are also palindromic numbers—the same whether they are read from left to right or right to left. An example of a palindromic number is 123321. Students have the opportunity to experiment with a technique that usually produces a palindromic number while they practice addition and collect some data for future discussion and analysis.

Musical Palindromes extends students' newly learned knowledge of music to a more creative realm—that of music writing. But this is music writing with a little twist—a musical piece that contains a palindromic sequence.



# \$1,000,000 LONG

#### TEACHER'S PLANNING INFORMATION

#### **Math Topics**

Numeration, estimation, computation with large numbers, problem solving, mathematical connections

#### **Active Learning**

#### Students will

- I. Estimate the length of one million dollar bills
- 2. Work collaboratively to develop problem-solving strategies
- 3. Measure one bill and compute the length of one million bills
- 4. Convert their results to appropriate units of measure
- 5. Use a map to determine the distance
- 6. Discover important landmarks within a circle of a determined radius

#### **Materials**

Rulers; dollar bills; maps; calculators; \$1,000,000 Long Worksheets

#### **Suggestions for Instruction**

Hold up a dollar bill and ask, "How long do you think one \$1 bill is?" If all of the responses are written on the blackboard, the estimates can be used to do some statistical analysis. Students can be asked for the range, the mode, or the mean. If they are ordered, the median can be found. Once some statistical analysis is done, students can be asked if they wish to change their estimates or not.

Then ask, "If we placed one million of these end to end, how far do you think they would reach?" At this point, students should not be given the use of calculators. After some discussion, place students in pairs and give each pair of students one copy of the \$1,000,000 Long Worksheet. Have each pair record its estimate on its worksheet in the space provided. To encourage mathematical reasoning, it is important for students to write a detailed explanation of where they could travel and how they calculated the distance.

 http://hypertextbook.com/facts/1999/Denene Williams.shtml contains information about not only the length of a \$1 bill but also its thickness—it is 1/10 mm thick.

#### **Selected Answers**

A dollar bill is approximately 6.25 inches long, so it can be used as a handy benchmark to help one estimate the length of objects. One million would be approximately 6,250,000 inches, or 520,833.3 feet, or 98.6 miles long.

#### INVESTIGATIONS: ESTIMATION, LARGE NUMBERS, AND NUMERATION

#### **Variation**

Give each group of students a map and have them find the area of a circle formed with a 98.6-mile radius and find all of the important cities or land-marks within that area. Also, if students go to the Web site cited previously, they can conduct an experiment to calculate how high a stack of one million dollar bills would be.

#### Writing in Math

- 1. How far do you think \$1 billion would reach? Explain your reasoning.
- 2. Now that you know the approximate length of a dollar bill, how might you use this information as a benchmark to help you estimate other distances?



## \$1,000,000 Long

#### Worksheet

Name		
Date	Class _	
reach if they were placed		how far one million one-dollar bills would would reach across a football field? Across Write your estimate here:
million bills would actu measure. Write an expla	ally reach. Be sure to express nation of your reasoning and o	and a calculator to determine how far ones s your distance using reasonable units o calculations in the spaces that follow. Use a ld you go? How did you figure that out?
How does this answer c	ompare with your initial estir	mate? How would you rate your estimate.
What other cities or land	marks fall within the calculate	ed distance?

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# HOW MANY STRIDES TO WALK AROUND THE EARTH?

#### **TEACHER'S PLANNING INFORMATION**

#### **Suggestions for Instruction**

The distance around the Earth is 24,902 miles or 40,074 km. This activity does not direct students to measure using customary units or metric units. This is not an oversight. It is possible to use either system of measurement, depending on curricular needs. Whether customary or metric units are used, students will need to convert miles to feet or inches or kilometers to meters or centimeters.

Begin the activity by asking, "How many strides do you think you would take if you were to walk around the earth's equator?" It is important for students to understand that it takes two steps to form one stride. Demonstrate a two-step stride. Ask students if they believe this is a normal stride. Students should see that any one stride cannot be considered "normal"; multiple strides (20 or more) need to be taken and the total distance divided by the number of strides to find the average length of just one.

Place students in collaborative groups of four, give them the materials they need, and have them proceed to solve the problem. When group averages have been calculated, bring the class back together to find the length of an average stride for the class. Class results can be analyzed to find the range of the data, the mean, median, and mode, any outliers, and so forth.

A very interesting book to supplement this activity is Kathryn Lasky's (1994) *The Librarian Who Measured the Earth.* It tells the story of Eratosthenes (circa 200 BC), an ancient Greek librarian, who figured out how to calculate the circumference of the Earth by using the angles formed by the sun's shadows.

#### **Math Topics**

Numeration, estimation, computation with large numbers, measurement, problem solving with large numbers, averages, mathematical connections, reasoning

#### **Active Learning**

Students will

- 1. Work in groups of four to solve this problem
- 2. Problem solve the length of a normal stride
- Accurately measure the length of their strides
- 4. Find the average or mean length of a stride for their group
- Compute the number of "normal" strides it would take to walk around the Earth
- 6. Combine their group's data with the class's data to facilitate statistical analysis

#### **Materials**

Metersticks or yardsticks (or tapes); copies of How Many Strides to Walk Around the Earth? Worksheet 1; calculators; overhead transparency of How Many Strides to Walk Around the Earth? Worksheet 2

- http://www.lyberty.com/encyc/articles/earth.html shows students how the circumference of the Earth can be calculated using the formula  $C = \pi d$ .
- http://www.guinnessworldrecords.com/content\_pages/record.asp? recordid=48612 tells the story of David Kunst who was the first verified person to walk around the world.

#### **Variation**

Have students compute the number of strides to walk to the moon (an average distance of 384,000 km or 239,000 mi).

#### Writing in Math

- 1. Why did your group use the mean length of your strides to solve the problem?
- 2. It would take a train traveling 100 kph (161 mph) about 99.5 days to reach the moon. How long would you estimate it would take (on the average) for you to walk there?



### **How Many Strides to** Walk Around the Earth?

### Worksheet I

Name	
Date	Class
Suppose you went on a long hike arc	ound the earth's equator. How many strides would it take?
walking with your left foot, when yo In your group, problem solve how yo the length of a stride for each memb that follows. Find the mean (or aver-	you travel when walking two steps. For example, if you start our right foot touches the ground, you have walked one stride ou might find a normal stride for each member; then measure per of your group and enter these measurements on the table (age) length of one stride for the members of your group. But what you will do to determine a "normal" stride.
Our Group Data	
Name of Person	Length of Stride

length computed from your group's experiment.

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# How Many Strides to Walk Around the Earth?

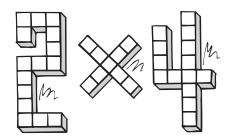
Worksheet 2

#### **Class Data Sheet**

Group	Mean Length of Stride
Mean Length of Stride for Class	

Is there a difference between the mean for the length of a stride for individual groups and the whole class? If there is, why do you think this occurred? Write your answer on the back of this page.

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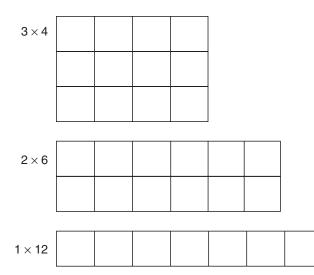
# RECTANGLES AND FACTORS

#### TEACHER'S PLANNING INFORMATION

#### **Suggestions for Instruction**

Place students into pairs, provide them with the necessary manipulatives, and give them time to find the factors of each of the first 20 numbers. Each pair of students will need about 30 tiles.

To demonstrate the activity, place 12 tiles on the overhead projector and ask for student volunteers to place these tiles into rectangular arrays. Some possibilities are  $1 \times 12$ ,  $2 \times 6$ , and  $3 \times 4$ . (For the purposes of this activity, a  $1 \times 12$  and a  $12 \times 1$  will be considered the same array.) The possibilities look like this:



#### **Math Topics**

Numeration, factors, area, prime and composite numbers, geometry

#### **Active Learning**

Students will

- I. Work in pairs for this activity
- 2. Use manipulatives to form rectangles
- 3. Understand that the sides of these rectangles are factors of the area
- 4. Discover the difference between prime and composite numbers

#### **Materials**

Buckets of square tiles (cardboard tiles can be used), Rectangles and Factors Worksheets, overhead tiles for demonstration

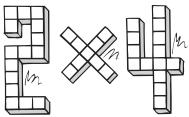
Students will discover that prime numbers have only one rectangular array, whereas composite numbers have at least two.

#### **Variation**

Students can be encouraged to find the prime factors of each of the numbers and, using combinations, find the integral factors. For example, the prime factors of 12 are  $2 \times 2 \times 3$ ; the combinations are  $2^0$ ,  $2^1$ ,  $2^2$ ,  $3^1$ ,  $2^1 \times 3^1$ , and  $2^2 \times 3^1$ . This activity gives them the opportunity to work with concrete materials and abstract concepts simultaneously.

#### Writing in Math

- 1. Explain how the number of rectangles formed by the factors of a number can tell you whether the number is prime or composite.
- 2. Some of the numbers between 0 and 21 have an odd number of factors. How would you describe these numbers?



### **Rectangles and Factors**

### Worksheet

	_		
Name			
Doto		Class	

**Directions:** Form rectangles using the number of tiles shown. Describe each of the rectangles in the space provided. Then record the factors for each rectangle.

Number of Tiles	Description of Rectangles (Length $\times$ Width)	List of Factors
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Write your observations about the size of the rectangles and their factors on the back of this page.

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#### ACTIVE LEARNING IN THE MATHEMATICS CLASSROOM, GRADES 5-8



# VENN DIAGRAMS: LCM AND GCF

#### TEACHER'S PLANNING INFORMATION

#### **Math Topics**

Numeration, number theory, reasoning, Venn diagrams

#### **Active Learning**

Students will

- Work in pairs to solve these problems
- 2. Find the prime factors of a pair of numbers
- Place the factors correctly in a Venn diagram
- Understand the relationship between the intersection of the sets and the GCF (greatest common factor)
- 5. Understand the relationship between the union of the sets and the LCM (least common multiple)

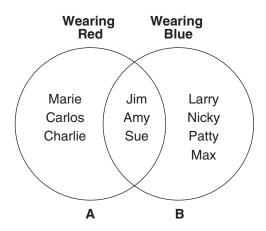
#### **Materials**

Venn Diagrams: LCM and GCF Worksheets

#### **Suggestions for Instruction**

This activity assumes that students understand how to find the prime factors of a number, perhaps by using factor trees. Venn diagrams are used in this activity to help students find the greatest common factor and least common multiple of two numbers.

Venn diagrams are named for the mathematician who developed them, John Venn. They have been used since the late 1800s. Venn diagrams are used to show the relationships between different elements in a set. For example, if we want to represent the set of students who are wearing red, blue, or both in class, our Venn diagram might look like this:



This diagram can be copied on the blackboard or on an overhead transparency, or one can be made using actual students. This simple diagram can be used to help explain the reasoning behind the placement within the diagram.

Some questions to ask students:

- 1. Who are all the students in this group? (This is the union of the two sets:  $A \cup B$ .)
- 2. Who are the students wearing both red and blue? (This is the intersection of the two sets:  $A \cap B$ .)
- 3. See how the circles overlap but not completely; why?
- 4. Where might we write the names of the students who are not wearing any red or blue?

After students understand the placement of members in a Venn diagram, give each pair of students a copy of the Venn Diagrams: LCM and GCF Worksheet. Read the directions with the students and be sure that they all understand how to find the prime factors of each of the numbers. Then explain how the numbers were placed in the Venn diagram shown on the worksheet.

Students can now work in pairs, following these steps: (1) choose two numbers, (2) find the prime factors of each number, (3) place the numbers correctly in the Venn diagram, and (4) find the union (LCM) and intersection (GCF) of the two numbers.

http://www.teach-nology.com/web\_tools/graphic\_org/venn\_diagrams/

This is a Web site that will allow the teacher to create Venn diagrams for student use.

• http://www.shodor.org/interactivate/activities/vdiagram/index.html

This is a wonderful interactive Web site that asks students where a particular item should be placed and then allows the student to know if the answer is correct or not. It includes questions about number theory, algebra, people, and so forth. It has very diverse offerings.

• http://www.stat.sc.edu/~west/applets/Venn.html

This interesting interactive site shades in two rectangles (A and B). Students get to choose from the following: A, not A, B, not B, A and B, A or B, not (A and B), not (A or B). The site also indicates the geometric probability of each of these events based upon the area of each rectangle.

#### **Variation**

Expand the activity to include 3-circle Venn diagrams that examine the LCM and GCF of three numbers.

#### Writing in Math

- 1. Describe the differences between these Venn diagrams: (6 and 8) and (8 and 24).
- 2. Can you think of two numbers where the circles would not overlap (have no intersection)?



# Venn Diagrams: LCM and GCF

Worksheet

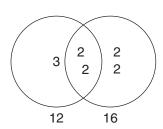
Name	
Date	Class

#### **Directions:**

- 1. Find the prime factors of your numbers.
- 2. When the two numbers share a factor, place that factor in the intersection of the two circles.

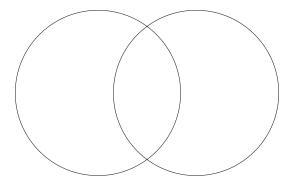
**Remember:** The intersection of the two circles is the GCF (greatest common factor). The union of the two circles is the LCM (least common multiple).

Example: Let's look at 12 and 16 The prime factors of 12 are  $2 \times 2 \times 3$  The prime factors of 16 are  $2 \times 2 \times 2 \times 2$  The intersection, 4, is the GCF The union, 48, is the LCM



My two numbers are \_\_\_\_\_\_

Their prime factors are \_\_\_\_\_



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# DESSERT FOR A CROWD

#### TEACHER'S PLANNING INFORMATION

#### **Math Topics**

Fractions, problem solving, connections

#### **Active Learning**

Students will

- Work in pairs to solve an open-ended problem
- 2. Use fraction concepts and skills to solve a real-world problem
- 3. Convert a recipe to feed a larger number of people

#### **Materials**

Dessert for a Crowd Worksheets I and 2 for each pair of students

#### **Suggestions for Instruction**

Discuss with students how pastry chefs need to convert recipes to feed different numbers of people. You can use an example of a recipe that makes a cake that serves 12. If there are going to be 50 people at a party, the chef could convert the recipe in the following way:

$$\frac{50}{12} = 4\frac{1}{6}$$

The chef needs to have between four and five cakes. To make sure there is enough dessert, the chef will need to have five times the amount of each of the ingredients in the recipe.

Give each pair of students a copy of the original recipe. Read the directions with them and make sure they understand appropriate measurements (for example, can you have half an egg?). Make sure students

agree on the number of students in the class. Suppose the class has 28 members. Students need to calculate the conversion factor, and that can be done in a number of ways:

An algebraic equation: 8n = 28; where *n* represents the conversion factor

A ratio and proportion:  $\frac{1 \text{ cake}}{8 \text{ people}} = \frac{n \text{ cakes}}{28 \text{ people}}$ 

A division problem:  $28 \div 8 = 3.5$ ; the conversion factor is 4.

Place the students into pairs and have them problem solve how they will convert the amount of ingredients needed to serve eight people to the amount of ingredients needed to feed the number of students in the class. After each group has developed its own strategy, have them convert the recipe (including the directions) to feed the entire mathematics class.

• http://kmiller.ecorp.net/recipe/ is only one of the Web sites that has a calculator to calculate changes that need to be made in a recipe to feed a

particular number of people. There are many other sites that will convert customary units to metric and metric to customary.

#### **Variation**

This recipe can be enlarged to feed the entire grade level or the entire school. The calculations become more difficult as the number of people to be fed is enlarged.

#### Writing in Math

- 1. Explain the procedures (the strategies) you used to enlarge the recipe to feed the entire class.
- 2. How do you think your strategies would change if you needed to convert the recipe to feed four people instead of eight?



#### **Dessert for a Crowd**

Worksheet I

**Directions:** The recipe for devil's food cake with marshmallow frosting will serve about eight people (each person will get 1/8 of the cake). Work with your partner to alter the recipe so that you can make enough cakes to feed the class. Be sure to rewrite the directions so that you will be making the correct number of cakes.

#### **DEVIL'S FOOD CAKE**

½ cup margarine 2 cups flour

1½ cups sugar I teaspoon baking soda

I egg ¾ teaspoon salt

2 egg yolks I cup milk

3 ounces unsweetened chocolate, I teaspoon vanilla extract

melted and cooled

Cream butter. Gradually add sugar and cream until light and fluffy. Add egg and egg yolks, one at a time, beating well after each addition. Add chocolate. Add dry ingredients alternately with milk. Add vanilla. Pour into two round 9-inch layer pans. Bake in a 350° oven for about 30 minutes. Cool and frost with marshmallow frosting.

#### MARSHMALLOW FROSTING

 $1\frac{1}{2}$  cups sugar  $1\frac{1}{2}$  teaspoons corn syrup  $\frac{1}{3}$  cup water 1 teaspoon vanilla extract

¼ teaspoon salt 16 (¼ pound) marshmallows, quartered

2 egg whites

Combine all of the ingredients, except vanilla and marshmallows, in the top part of a double boiler. Beat for 7 minutes, or until stiff peaks form. Remove from heat and add vanilla and marshmallows.

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### **Dessert for a Crowd**

Worksheet 2

Name	
Date	Class
	partner to figure out the correct quantities of ingredients for thi ough for every student and a piece for the teacher! Use the space is.
	DEVIL'S FOOD CAKE
cup margarine	cups flour
cups sugar	teaspoon baking soda
egg	teaspoon salt
egg yolks	cup milk
ounces unsweetened	teaspoon vanilla extract
chocolate, melted and cooled	
well after each addition. Add	ar and cream until light and fluffy. Add egg and egg yolks, one at a time, beating chocolate. Add dry ingredients alternately with milk. Add vanilla. Pour into ch layer pans. Bake in a 350° oven for about 30 minutes. Cool and frost with
	MARSHMALLOW FROSTING
cups sugar	teaspoons corn syrup
cup water	teaspoon vanilla extract
teaspoon salt	marshmallows, quartered
egg whites	
	except vanilla and marshmallows, in the top part of a double boiler. Beat so form. Remove from heat and add vanilla and marshmallows.

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# 5 × 5 PUZZLE CENTS

#### TEACHER'S PLANNING INFORMATION

#### **Math Topics**

Decimals, problem solving

#### **Active Learning**

Students will

- I. Use decimal skills to solve a puzzle
- 2. Work to find multiple solutions

#### **Materials**

Scissors,  $5 \times 5$  Puzzle Cents Worksheets, calculators (if necessary)

#### **Suggestions for Instruction**

Students can work alone or in pairs. After cutting out the coins on the bottom of the worksheet, allow students time to solve the puzzle. By moving the tokens around, students will find it easier to try different possibilities. Sums give a clue to the numbers that belong in the blanks.

#### **Selected Answers**

A possible solution is shown below. There are other ways to solve this puzzle.

10¢	5¢	50¢	25¢	25¢
10¢	5¢	5¢	10¢	10¢
5¢	25¢	1¢	25¢	25¢
50¢	10¢	50¢	5¢	50¢
1 <i>¢</i>	50¢	1¢	1¢	1¢

#### **Variation**

Students may be given the option of using calculators to solve the puzzle. Have students create their own puzzle for the rest of the class. Students can also work on magic squares where the sum of the numbers in each column, row, and diagonal add up to the same number. This is an example of a decimal magic square.

**Directions:** Use the numbers 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, and 1.8 to make a magic square with a sum of 3.0. There are many solutions to this problem but one of them is shown below.

#### INVESTIGATIONS: ESTIMATION, LARGE NUMBERS, AND NUMERATION

0.4	1.4	1.2
1.8	1.0	0.2
0.8	0.6	1.6

#### Writing in Math

- 1. Explain why *lining up the decimal points* when adding decimals and *finding common denominators* when we add fractions speak about the same mathematics concept.
- 2. Explain whether the sums at the end of the rows helped you solve the problem. If you used another strategy, explain how it helped you solve the problem.



\$0.76

\$0.95

### **5 × 5 Puzzle Cents**

### Worksheet

	Class	
is $5 \times 5$ grid so th	at the totals of each row and	
		\$1.15
		\$0.40
		\$0.81
		\$1.65
		\$0.54
	in each square, ar	in each square, arrange five pennies, five nickels is 5 × 5 grid so that the totals of each row and v and under each column.

\$1.07

\$0.66

\$1.11

### $5 \times 5$ Puzzle Cents—Worksheet (Continued)

	32:00	Ar cord	The cord	ALCON DE LA COLONIA DE LA COLO
		The same of the sa		
DIV				
PIER DEST	TER DIST	THE THE PARTY OF T	TER DOS	THE DAY OF THE PARTY OF THE PAR
ATES	ATES ON THE STATE OF THE STATE	ATES ON THE STATE OF THE STATE	ATES ON THE PROPERTY OF THE PR	THE SOLL STATE OF THE SOLUTION OF TH

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#### ACTIVE LEARNING IN THE MATHEMATICS CLASSROOM, GRADES 5-8



# CHOCOLATE CHIP COOKIES

#### TEACHER'S PLANNING INFORMATION

#### **Math Topics**

Computation, problem solving, fractions, decimals, connections

#### **Active Learning**

Students will

- I. Work with a partner to solve the problem
- Compute the total cost of each of the ingredients
- Compute the total cost of the cookie batter
- 4. Compute the cost of one cookie
- Compute the profit and percentage of profit

#### **Materials**

Chocolate Chip Cookies Worksheets, calculators

#### **Suggestions for Instruction**

Discuss with students the mathematics of cooking! Ask them what math they think a chef or caterer might need to know. Students might say that chefs need to convert recipes or determine how much to charge for a meal. After students have had a chance to examine the worksheet, discuss the problem and why it is important to find the total cost of each of the ingredients in a recipe, regardless of the quantity needed. Students may need assistance in converting fractions to decimals and interpreting the answers. The cost of baking soda and salt is less than  $1\phi$ ; this may pose difficulties for some students. These problems are challenging to solve because they combine the multiplication of fractions and decimals. In the first problem, students are asked to multiply the mixed number 4½ by the decimal 0.32. While it is possible for students to change the  $32\phi$  to a fraction, they will most likely want to change the  $\frac{1}{2}$  to 0.5.

Have students work with a partner. After they find the total cost for each ingredient, they are asked to find the cost per cookie.

#### **Selected Answers**

The total cost for the ingredients is about \$30.75; the cost per cookie is about  $13\phi$ ; Depending on how the pairs rounded, the profit is about 290%.

#### **Variation**

Consumerism issues (i.e., percentage of profit) may be of great interest to students at this level. Interesting problems involve finding the cost per pound of cosmetics, perfume, or other very highly priced items. For example, suppose ground beef selling for \$1.89/lb is used to make a ¼-pound hamburger. The cost

for the meat is about  $46\phi$ . If the bun costs  $10\phi$  and the rest of the ingredients add another  $4\phi$ , the total cost for the hamburger is about  $60\phi$ . If the restaurant sells it for \$2.49, the percentage of profit is over 300%. But what other expenses does a restaurant owner have besides the cost of the ingredients used?

#### Writing in Math

- 1. Explain how there can be a percentage greater than 100%. Give an example.
- 2. What would happen if a  $25\phi$  cookie was sold for the price in your example?



What is the percentage of profit?

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# **Chocolate Chip Cookies**

#### Worksheet

ate		Class	
Ingredient	Amount Needed for Recipe	Cost per Unit	Total Cost for Each Item
Margarine	4½ lbs.	\$0.32/lb.	
Creamed Shortening	4½ lbs.	\$0.48/lb.	
White Sugar	8½ lbs.	\$0.31/lb.	
Brown Sugar	7 lbs.	\$1.01/lb.	
Eggs	40	\$0.08/ea.	
Vanilla (imitation)	½ cup	\$0.40/cup	
Flour	16 lbs.	\$0.15/lb.	
Baking Soda	6 tablespoons	\$0.013/tablespoons	
Salt	6 tablespoons	\$0.0017/tablespoons	
Chocolate Chips	9 lbs.	\$2.56/lb.	
TOTAL COST			

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