Chapter 1

A Context for Solving Quantitative Problems

THE PROBLEM

Phil is an academic counselor at the local community college where he's been a member of the staff for several years. He hopes one day to work in the office of institutional research, perhaps as its director. It's been years since he took a class but he recognizes that some of the questions that he and others ask about how they're doing at the college require some ability with quantitative analysis. He picks up a statistics text in the bookstore, leafs through the pages, and wonders whether he's ready to pursue an advanced degree.

QUESTIONS AND ANSWERS

❑ What does this book have to do with my career as an educator?

From kindergarteners to graduate students, today's learners belong to the most scrutinized and indeed the most tested generation of students. Many of the questions asked about the quality of their education and about their educational progress require some quantitative analysis. This book is designed to help with just such problems.

❑ How much of a mathematician will I need to be to navigate this textbook?

The answer is not a mathematician at all in the usual sense of that description. A little introductory algebra will help but you will need nothing beyond. It's far more important to think logically than it is to have an extensive background in mathematics.

❑ Since computers and software are available for statistical analysis, do I have to perform the calculations myself?

For two reasons, the answer is yes. Those who can work solutions understand the analysis more readily than those who cannot, and it's actually quite satisfying to complete a solution longhand, although you may have to trust me on this for now.

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About Statistics

When things are important to us, we often create a record of them. When a piece of music is particularly moving, a meal is outstanding, or a friendship is unusually meaningful, we at least create memories that serve as a more-or-less permanent record of the experience. Some of the other things that we wish to remember are important not because they involve the emotions that the foregoing evoke, but because they are a record of the choices made in the past and may become elements in the decisions that are yet to come. This seems particularly likely at a time when educators at all levels are scrutinized more carefully than perhaps they ever have been. Ours is an age of educational accountability.

Numbers offer a great economy in record keeping. Although certainly there are some things that can't be readily reduced to numbers, this book is about describing and analyzing things that can. Some numbers are nothing more than convenient labels to indicate a category to which an individual belongs. This is the case when summaries are made of something like students' academic majors, and for convenience, the majors are numbered. At other times, the number indicates how much of some quality an individual possesses, as with a verbal aptitude gauge where higher scores indicate that the individual measured possesses more of, or a higher level of, some characteristic than one with a lower score. In either example, the numbers allow one to reduce what could be lengthy verbal descriptions to a relatively compact record. Whether we're gathering information about a second language speaker's reading comprehension, the dropout rate in a particular school district, or the number of units for which entering freshman students typically register, someone is relying on their ability to quantify several important outcomes. Efforts to discover more effective teaching, curb the dropout rate, or evaluate trends in student registration often depend on educators' abilities to manipulate and analyze quantitative data.

This is the reality that Phil faces in the chapter introduction. He, and all educators and decision makers, needs to be able to answer questions about differences and about relationships, about the way things are and how they might be improved. Questions about the average age of entering freshmen, or the changing attitudes of entering freshmen, or about whether there have been changes in students' level of preparation for college-level study over time may involve quite intricate quantitative analyses. Equipping readers to deal with such issues is the point of this book.

Regarding the Math

To be straightforward about a couple of things as we begin, note that by its nature statistical analysis is mathematical and a good deal of basic math is involved in some of the things we will do as we progress, but it is *basic* math. Because the issues educational decision makers face are complex and involved doesn't always mean that the analytical techniques used to confront those issues need to be.

Does that sound inconsistent? Perhaps only continuing in the book will be convincing, but in a number of years of teaching statistical analysis, your author has come to two conclusions:

1. Students often have very little background in quantitative analysis generally, and little experience with mathematics specifically beyond an introductory algebra course.
2. The lack of mathematics needn't be an impediment to statistical analysis and decision making.

Perhaps like the aspiring institutional researcher in the chapter introduction, that last (first?) algebra class was as long ago as somewhere in secondary school, but he, and you, can flourish nevertheless.

By the way, if the reader has *more* than a little algebra that's terrific, but that hasn't been assumed. The more advanced students among us will just need to be a little patient during the early chapters while those who have had less math, or a longer interruption, get up to speed.

If it *has* been a while since formal mathematics coursework, one of the more important things to do by way of beginning is just to review things like the order in which multiple mathematical operations must occur. Many of the mistakes that students make when there are multiple calculations required stem from forgetting that, in order, the student works

- in parentheses first,
- then with exponents,
- then with multiplication and division, working from left to right, doing whichever comes first, and
- then addition, and subtraction, again working the leftmost problem first.

If "**p**lease **e**xcuse **m**y **d**ear **A**unt **S**ally" still helps one remember, great. If it doesn't, find something else to use as a guide because some of the formulae involve several calculations. It's important to be clear about whether we subtract or divide first when the formula calls for both.

For students whose earlier forays into some sort of quantitative analysis were, well, modest in outcome, a course like this can be about redemption. With some persistent study and practice, even those whose prior experiences prompt trepidation at the prospect of a statistics class can excel.

Answers the Old-Fashioned Way

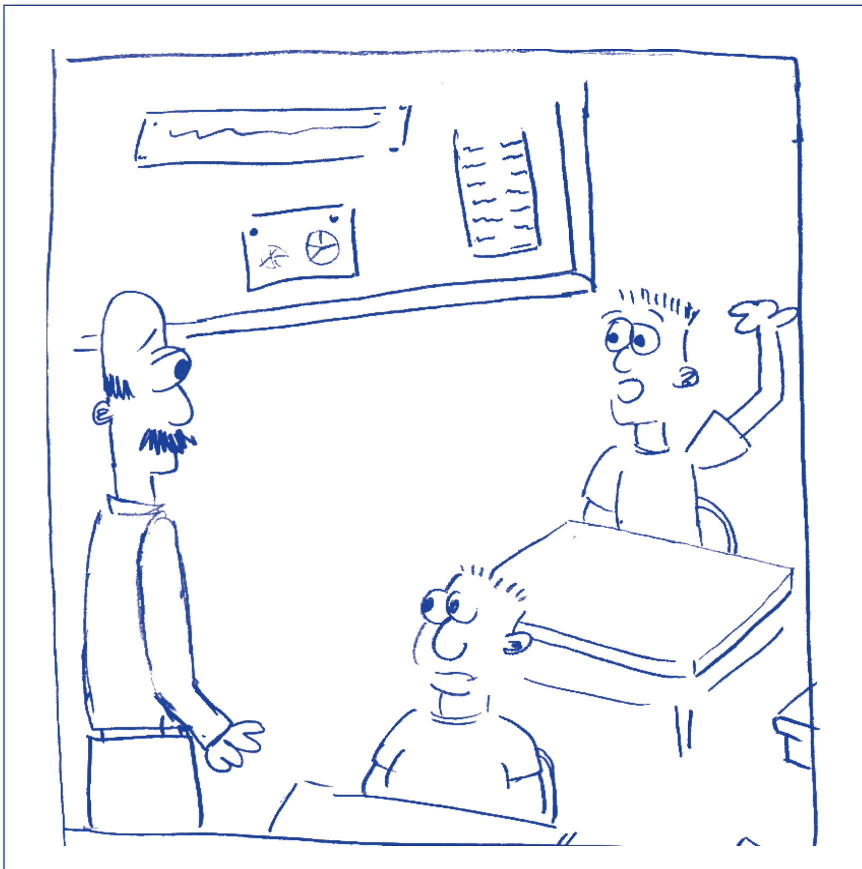
The truth of the matter is that in the present age, beyond balancing your checkbook (and perhaps not even that), relatively little quantitative analysis is carried out without using a computer. Sometimes there is just too much information to deal with to make avoiding the computer practical. Even when there aren't mounds of data to absorb, computers are very convenient. Business-oriented spreadsheets such as Microsoft's Excel, to say nothing of dedicated statistical packages like SPSS, have built-in statistical procedures that address most of the problems we'll tackle in an introductory statistics textbook. Indeed, statistical packages provide analyses of extremely complex and involved problems. And yet there is a good deal of emphasis in this book on hand calculation. Why? The question deserves some attention.

It isn't just the aspiring institutional researchers such as Phil who wish to answer questions that involve some data analysis. We all have questions that require some "number crunching," but when we rely entirely on a computer to do the work a problem emerges. In an effort to be user-friendly, software developers have made generating solutions very easy, so easy in fact that you and I can run elaborate statistical procedures without really understanding whether we have satisfied the assumptions that the procedure requires, or whether we even understand the implications of the output. The user is not only the beneficiary, but sometimes also the victim of the software developer's ability to make complex analyses accessible.

To counter this problem, we'll do some pencil-and-paper work. With the exception of some of the procedures in Chapter 16, we'll work each new type of problem longhand before leaning on the computer for answers. The beauty of longhand calculations is that every step is visually discrete. Instead of just trying to understand what the solution means, we can consider why each calculation is made and also note its impact. How and why everything is done is usually more transparent working longhand than it is when peering at computer output. If the "it's good for you" argument makes all of this sounds like eating oatmeal when you'd really rather have waffles, longhand calculations have an ancillary benefit. Working solutions can actually be quite satisfying; they can prompt quite a feeling of accomplishment.

It's Greek to Me!

A little statistical notation is going to help us. Although sigma (Σ), rho (ρ), mu (μ), and other Greek symbols can be intimidating at first glance, there's no need to worry. We're making no forays into Sir Isaac Newton's calculus. For us the symbols are components of a mathematical shorthand. They're



SOURCE: Created by Suzanna Nielson.

"When do we learn to calculate the over and under on football games?"

symbols for values we'll either calculate without much difficulty or abbreviations for mathematical procedures you probably already know how to complete. They really aren't going to be a problem. Each symbol will be explained before it's pressed into service. Besides, think of the mileage you'll get from casually mentioning to your colleagues that $\mu = 85.437$ for eighth-grade language arts students on the district's benchmark math test!

DIFFERENT KINDS OF STATISTICS

- **Descriptive statistics** summarize a data set.
- **Inferential statistics** reveal the larger group through the smaller group's characteristics.

If he is to work in his college's Office of Institutional Research, Phil will need to be able to both describe and analyze data. In fact, describing and analyzing define the bulk of statistical tasks. The **descriptive statistics** we'll begin working with in Chapter 2 allow one to summarize the characteristics of a data set. Sometimes the descriptive task employs graphs and figures to represent the data. Developing the tools for these show-and-tell descriptive tasks are the subjects of Chapters 2 and 3.

Beginning with Chapter 5, and then carrying on through the balance of the book, the focus will change from describing data to drawing inferences about data. There we will learn how to understand large groups by examining smaller groups. That's the domain of **inferential statistics,** and it allows anyone dealing with quantitative data to have a great economy in her or his analysis. When we observe certain conventions about the way the smaller group is formed, it can tell us a good deal about the nature of the larger group, which brings us to the next topic.

SAMPLES AND POPULATIONS

- A **population** is all members of a defined group.
- A **sample** is any subset of a population.

Statistical analysis involves two kinds of groups, all possible members of any specified group that defines a **population**, and any subset of the population, which is called a **sample**. When noting just above that one can view the larger group through the lens that a smaller group provides, we were talking about understanding the population by analyzing the sample. If we're careful about the way the sample is selected, we can come to know a great deal about the aptitude for college study among *all* freshman students (the population) by examining the aptitude that a smaller number (the sample) possess, as long as they're representative. The reader will always know which group we're talking about because we'll use Greek letters to indicate the characteristics of populations, and more familiar Roman letters for samples.

The symbol μ (mu) mentioned above is the symbol for the arithmetic average (the mean) of some characteristic in a population, verbal ability for example. As long as he's referring to just his institution, all students at Phil's school represents a population. If the mean age for all students in that group is 24.545, he could indicate that as $\mu = 24.545$. The corresponding symbol for the mean of the sample is *M,* which although not universally used, is increasingly common. (It used to be common to use x-bar, an ex with a bar over it, to signify the mean, but many of the major research journals have adopted *M*.) If instead of everyone, Phil selects a sample of 30 students from his institution and determines their average age to be 22.916, he would represent that value as $M = 22.916$.

The Connection Between Quantitative Analysis and Research Design

Educational decision makers have little interest in **constants** precisely because they don't vary. How interesting would it be to study the weather if everyone had the same weather and it was the same every day? **Variables**, conversely, have that name because they have different manifestations. **Qualitative variables** include such characteristics as an individual's gender, ethnicity, political affiliation, or school site. Differences in variables like those are indicated not by the amount of the characteristic one possesses, but by the category to which an individual belongs. We may use numbers to code the site at which one teaches, for example, but as is the case with all such variables, the number only indicates the category and has no mathematical significance.

- **Constants,** which never vary, hold little analytical value.
- **Variables**, however, are interesting precisely because they have different manifestations.

By contrast, for **quantitative variables** there are measurable differences in whatever is gauged. For characteristics such as age, spelling ability, or intelligence, differences in the amount of what is measured are indicated by the numbers that are assigned. Examining something like achievement differences between primary English speakers and English learners involves both a qualitative and a quantitative variable. The group into which one falls (primary English or English learner) is a qualitative variable. The individual's score on a reading test is a quantitative variable.

- **Qualitative variables** differ by category rather than by amount, but there are measurable differences in **quantitative variables**.

So constants hold no allure for us, but variables pique our curiosity. We all have questions and perhaps theories about why things vary. When we put together a formal plan for gathering and analyzing data so that we can answer those questions, we have developed a **research design**. Part of the purpose of a research design is to specify what the researcher believes to be the

- **A research design** is a formal plan for gathering and analyzing data.

- The variable thought to affect another is called the **independent variable**.
- The variable influenced is the **dependent variable**.

relevant variables. We'll deal with this in more detail in Chapter 6, but when one variable is thought to have an effect on another, that antecedent variable is termed an **independent variable** and the one upon which it is judged to have an effect is called the **dependent variable**. If homework is thought to have a facilitative effect on students' grades, homework is an independent variable and grades, the dependent variable. We raise the issue now because part of the point of a book like this is to put us in a position to answer such questions. Although this isn't a research design text *per se*, the topics we will study are all topics intimately related to design issues.

TESTING ISSUES

For any K–12 educator and to some degree also for higher educators, there seems to be little refuge from testing. If Phil is successful in his pursuit of the Office of Institutional Research, managing and analyzing test data will be a significant part of his task. With questions about student admissions, students' disabilities, mandated educational goals, and so on, testing issues pop up everywhere. As it turns out, many of those testing issues have important statistical components, and the tools we can develop in a course such as this will help us to understand the issues better. For that reason, a section on Test and Measurement Topics has been included. In Chapter 15, we'll use some of the descriptive and analytical procedures developed earlier in the book to examine issues such as score reliability and detecting testing bias.

Historically, test and measurement issues were almost exclusively the domain of the testing experts. Certainly testing issues sometimes require a highly technical and esoteric set of skills and knowledge, but testing issues have becomes so pervasive that educators who develop some ability with analyzing testing data are in a unique position to bring about improvement. Part of what drives the testing movement is the realization that it isn't enough to deliver curriculum and instruction. One must be able to document how well it was done. Educational testing has implications for educators and learners that are too important for the educators not to be more directly involved, so we'll develop some statistical tools that will allow us to tackle at least some of the fundamental testing questions.

CHAPTER ORGANIZATION

Just as was the case for this chapter, each of those that come after will begin with a *Problem* to provide a context for the material we're going to cover, accompanied by some questions and tentative answers to focus the reader on particular issues that an educator is likely to face. Periodically in the chapter, text boxes titled "Key Terms" such as those in this chapter will provide brief definitions of the technical terms that are introduced in each chapter. All of those concepts and definitions appear again in a Glossary at the back of the book.

All of the procedures we'll explore have applications beyond those which are suggested in the chapter. That's part of their value, of course. Although there are examples for each procedure in text and, frequently, *Another Example* thrown in for the value of the practice, there are also occasional boxes titled *Through a Wider Lens* to suggest a the broader application for the concepts. Study them for the suggestions they make about the more general uses there are for what we're doing.

Most of the chapters introduce new formulae. Each one is numbered and explained, and then listed at the end of each chapter. The end-of-chapter listing also repeats a brief explanation of each formula's purpose.

Except for this first chapter, each chapter also ends with a list of practice problems to supplement those worked longhand earlier in the chapter. According to the usual practice, solutions for half of the end-of-chapter problems are provided in a section at the end of the book.

Software and Hardware

With all of the matter of fact about the importance of working longhand solutions, even your author, who is very fond of calculating paper-and-pencil solutions, recognizes that it isn't practical with large data sets. The idea is to get you comfortable with computer applications as well, which brings us to SPSS, the "Statistical Package for the Social Sciences."

In the interest of student familiarity, many of the longhand problems in the chapters also have output from SPSS solutions. At the end of the book, there's an Appendix for SPSS procedures. It provides a step-by-step sequence for completing the problems worked in the chapters.

Several dedicated statistical software packages are available that will handle the problems we'll take up, and people who do a great deal of analysis probably each have their own favorite. However, among educators and social scientists generally, SPSS may be the most popular, which is one reason why a version of this book is available with a trial version of that software. Like many software publishers, the versions change from time to time, but don't be too concerned about obsolescence. The commands involved in the analyses and the appearance of the output have changed very little in the several most recent versions of the software.

One of the educational decision maker's assets is a hand calculator. Many of them have statistical functions built in, and some are very sophisticated in the solutions they can generate. Your instructor will probably have a recommendation about the particular type she or he wishes you to use. In an age of inflating prices in many areas, happily for us, electronic devices are relatively inexpensive. Your local discount supermarket probably has all the calculator you will need for this class for less than $15. Get one, and develop some familiarity with how the memories work and how to access the statistical functions. The more elaborate graphing calculators will also perform the statistical functions we need, but we aren't going to use the graphing function, and those more expensive tools probably represent calculator overkill for an introductory statistics class.

STUDYING FOR STATISTICS

Without wishing to sound paternal, a comment on how to study statistics seems appropriate. Part of the beauty of language arts instruction is that learners are obliged to practice the content whether or not class is in session. Ordinary day-to-day communication almost makes it impossible *not* to generalize much of the language learning. Because statistical analysis isn't nearly as commonplace as ordinary language communication, it doesn't offer the same advantage. Many opportunities exist to use statistics in the normal course of events to be sure, but we have to look for them. To minimize that limitation it's important to think about the material when *not* in class, which yields (drum roll, please) a rationale for homework. Doesn't advocating homework in a book for educators have a satisfying consistency? The work *out* of class is about more than practice. It's very much about learning to generalize new abilities beyond the classroom environment.

Between the reading and the homework, this material will take some time. As a practical matter, an hour each day during the week is going to pay greater learning dividends than a marathon the night before class. The statistical analyses we're going to tackle here aren't particularly difficult, but they may be different from the way you usually think and analyze problems. The kind of logic we're trying to cultivate seems to come more readily when we work at it often than when we occasionally work for long sessions.

And Finally

The Phil referred to in the "Problem" at the beginning of the chapter is intended to appeal to each of us, of course. It isn't that we necessarily aspire to head some institution's Office of Institutional Research, but the fact is that, like Phil, some of the really pressing questions every educator faces require some understanding of quantitative analysis. Developing solutions to those problems often doesn't require highly sophisticated mathematics. The skills and procedures we need to be effective in our educational careers are within the reach of the typical university student. To develop the grounding we need to be successful, we'll do some things that may seem a little old-fashioned, like producing longhand calculations that could more quickly be done by computer, but it's all part of the plan for your progress. Be clear about the overarching goal. It isn't that you will perhaps "weather," or survive the course, but that you will succeed, and in grand fashion. So let's begin in earnest.

STUDENT STUDY SITE

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