CHAPTER 1

TEACHERS AS MATHEMATICIANS

Aims

By the end of this chapter you should:

- be aware of factors that are specific to effective mathematics teaching;
- become aware of factors that may affect your teaching of mathematics;
- have identified your response to mathematics;
- have identified incidents and experiences in your autobiography that may have affected your response to mathematics.

Introduction

In the United Kingdom state school teachers are expected to teach the full curriculum for children up to age 11. There are some exceptions such as in those areas that operate a 'middle' school system for children from the age of 8 or 9 years, where some specialist teaching takes place. Some schools have an element of specialist teaching but it tends to be for Foundation subjects such as music and Physical Education in which it is standard

to acknowledge some teachers may lack sufficient subject knowledge. However, in core subjects such as mathematics such a view would appear not to be acceptable for someone with Qualified Teacher Status (QTS). Just as we would not assume that teachers are artists or have specialist skills to teach art, so we assume that those teaching in the primary age range do not require specialist skills to teach mathematics. The generalist nature of teaching in the first years of education has many benefits for children and teachers in terms of pastoral care of children and opportunities to make links between subjects. In recent years, however, these general assumptions have been questioned in terms of the challenges faced by the generalist in subject knowledge, curriculum organisation and delivery.

Particular concern has focused on mathematics and its falling ranking in relation to international comparisons. Data from the Trends in International Mathematics and Science Study (TIMSS) and, more recently, in the Programme for International Student Assessment PISA study by the Organisation for Economic Cooperation and Development (OECD) indicates that the UK does not compare favourably with other countries.

Beliefs and their effect on learning and teaching mathematics

Where would you consider you are on the following scale in terms of your feelings about mathematics? Is it a subject you love or hate, or maybe your feelings are not so extreme and fall somewhere midway?

10 …….. 0

Mike Askew led a research project by King's College, London that was sponsored by the Teacher Training Agency (TTA) (Askew et al., 1997). The

aim was to investigate the distinctive characteristics of effective teachers of numeracy. Effectiveness was measured in a test administered to the pupils of the teachers in the sample at the beginning of the Autumn term and the end of the Spring term and the average gain calculated. This produced an indicator of teacher effectiveness.

Using data from case studies it emerged that teachers held certain individual sets of beliefs about mathematics. Such beliefs were found to be fundamental in influencing the way in which they taught. The beliefs were categorised in the following way.

- *Connectionist* – beliefs based around valuing pupils' methods and teaching strategies with an emphasis on establishing connections within mathematics.
- *Transmission* – beliefs based around the primacy of teaching and a view of mathematics as a collection of separate routines and procedures.
- *Discovery* – beliefs clustered around the primacy of learning and a view of mathematics as being discovered by pupils.

(Askew et al., 1997)

One of the key findings was that effective teachers had connectionist beliefs. They made connections between different ideas and knew how to select and use effective and efficient strategies for calculation. They believed that their pupils could become numerate and that pupils develop strategies and connections between ideas or networks by being challenged to think by use of explanation, listening and problem-solving. There was little correlation between the effective teachers in the sample and their qualifications. However, there was evidence that they had undertaken mathematics professional development over an extended time.

Apart from the research led by Mike Askew, there has been considerable research carried out concerning beliefs about mathematics in both the adult population as a whole and more specifically in teachers of mathematics. In the UK alone there is a large body of evidence drawn from research over the last thirty years ranging from government reports, most significantly Cockcroft (1982) and Williams (2008), to research with teachers and those involved with education such as Buxton (1981), Haylock (2010), Jackson (2008), Boaler (2009) and Swan and Swain (2010). A brief overview of the research carried out may give you a sense of the significance of beliefs in the teaching of primary mathematics.

Over thirty years ago, Laurie Buxton, a mathematics adviser for the now defunct Inner London Education Authority, carried out research to investigate the aversion to mathematics that he was encountering among those involved with education. This group included a significant percentage

of teachers who were otherwise high achievers and successful in their chosen career. The result was a series of detailed case studies based on individual interviews, group work and discussions that still have resonance today. Although the research was carried out with a small sample of nine adults, the fact that Buxton was seconded for a year enabled him to have time to work with the group in depth. He found that their emotional response was very important to learning mathematics and using it in an educational context.

Participants spoke of mathematics in terms of terror. When asked about their feelings towards mathematics phrases used ranged from worried to panic to the extent that one participant stated '. . . whenever I think back to it [mathematics], it's always that dreadful numbing panic' (Buxton, 1981). Buxton worked closely with Richard Skemp, a mathematician, educator and psychologist. He designed mathematics workshops to enable participants to develop confidence, which he believed to be fundamental to learning. Basic to the strategies Buxton used was the interaction of the individual's emotions with mathematics. He found common feelings about mathematics among the group such as a belief that mathematics was a collection of incomprehensible rules and facts to be remembered that mainly involved computation. He acknowledged the validity of these perceptions as they came from the individuals' experience. He believed that the *experience* of learning mathematics had to be changed if mathematics was to be seen as a comprehensible subject with interconnecting relationships.

The effect of emotion on mathematics was confirmed in the Cockcroft Report (1982) by surveys it commissioned. While acknowledging that many people coped with the demands of mathematics, it was found that among the adult population, mathematics induced feelings of anxiety, helplessness, fear and even guilt among interviewees and 'No connection was found between the extent to which those interviewed used mathematics and the level of their educational qualifications . . .' (Cockcroft, 1982: para. 2.20).

Despite the detailed recommendations of Cockcroft to address the problem in the intervening years, Ian Thompson found, when working with teachers on mathematical activity, they responded similarly to those cited in Cockcroft (1982) with the same feelings of panic, anxiety and guilt (Thompson, 2003). Similarly, Derek Haylock has considered the emotional response to mathematics with teaching students. In the fourth edition of his book *Mathematics Explained for Primary Teachers* (Haylock, 2010) he states 'Even well-qualified graduates feel insecure and uncertain about much of the mathematics they have to teach' (Haylock, 2010: xii).

Elizabeth Jackson carried out a literature review of mathematics anxiety and subsequently investigated the situation among primary teacher trainees

(2008). She suggests that mathematical anxiety exists to the degree that the ability to do mathematics is strongly influenced by people's attitudes rather than their cognitive skill.

Jo Boaler has carried out extensive research into mathematics teaching and learning in the USA and UK. Having completed longitudinal studies she has identified what she believes is wrong in classrooms today in terms of children's mathematical experiences. She found that belief was significant in forming ideas about mathematics. Some teachers believed that success in mathematics was a sign of general intelligence and that it was a subject that some people can do and others cannot. Such ideas have the potential to be harmful to children as it makes them feel 'helpless and stupid' (Boaler, 2009: 1–2).

More recently, Malcolm Swan (Swan and Swain, 2010) has undertaken a number of research projects into the professional development of numeracy teachers with post-16 learners. Although relating to teaching those beyond statutory education the focus still relates to mathematics teachers. Teachers investigated eight research-based principles for teaching:

1. Build on the knowledge that learners bring to the session.
2. Expose and discuss common misconceptions.
3. Develop effective questioning.
4. Use cooperative small-group work.
5. Emphasise methods rather than answers.
6. Use rich mathematical tasks.
7. Create connections between mathematical topics.
8. Use technology in appropriate ways.

Existing beliefs and practices were recognised and alternative practices were offered and considered. The teachers were encouraged to adopt new practices and reflect on their experiences and beliefs. Results suggested that many of the teachers' practices and beliefs were significantly affected. Their practice became less transmission-oriented and teachers began to create collaborative learning environments where students were challenged to confront difficulties and take on more active classroom roles.

Assessing your beliefs

Initially, the findings of the research cited above may seem somewhat negative in terms of teachers and the teaching of mathematics. However, it does give an insight into four key issues that can be identified from the research.

- *The subject of mathematics can be seen to hold a level of mystique for a significant number of the adult population, including teachers.*

Mystique is a characteristic that can mean charm and magic to some yet a level of impenetrability to others. It would seem that the teacher's role is to enable children to engage in the subject and so find the magic it can yield.

- *Adult mathematics qualifications and cognitive ability do not necessarily correlate with effective mathematics teaching.*

Such findings may seem surprising but consideration of teaching you have received in the past may help clarify this point. You may remember being taught a subject or a skill by someone who was obviously an expert in their field but could not impart their knowledge in a way that made it accessible to you. Conversely, you may remember being in a similar situation but with someone who enabled you but was not an expert in their field. Awareness of how you were enabled can act as a starting point to making sense of generic teaching skills and lead on to relating and developing these skills in mathematics.

- *Beliefs about mathematics are fundamental to engagement and enjoyment with the subject.*

Feelings towards mathematics will come from a wide spectrum of possible responses from a complete lack of confidence in the subject in terms of personal understanding and teaching, to having strong confidence in the subject. How you feel about a subject can be affected by a range of factors. If you think back to your childhood, your feelings towards subjects may have been influenced by achievement, your teacher, fellow classmates, the setting or a combination of such factors. Some factors can be less direct such as health or family issues that can lead to children finding it hard to concentrate or are unwilling to risk failure. As a teacher of mathematics, a positive feeling towards the subject is likely to enhance its teaching. Indeed, enjoyment of the subject was seen as a required competency in one of the earlier versions of the TDA Standards for Qualified Teacher Status (QTS): 'enjoy mathematics so that they can teach it with enthusiasm' (DfEE, 1998: Section C of Annex D). Enjoyment was found to be a factor in teaching judged as 'excellent' by Askew and his team when they carried out research (Askew et al., 1997).

- *Assessment of beliefs leading to active intervention can yield positive results for the teacher, their teaching and learning of children.*

One of the heartening findings of the research discussed, is that active intervention through mathematics courses in teacher training programmes and Continuing Professional Development (CPD) can enable teachers to understand why mathematics holds a mystical quality and how teachers can enable a new generation to access its power. Teachers have the opportunity to become learners again. However, it is not the same as returning to childhood experiences of learning, although the experience may evoke memory of such experience. The difference is that in this new role of acquiring knowledge about mathematics, the learner will have the experience of past learning and teaching to call upon. All teachers have the potential to improve their teaching but often making a change is hard as there is a move from the familiar to the unknown. Additionally, making a change can initially have a negative effect on teaching as the mechanics of the change may feel unnatural. For example, if the aim is to develop a more interactive style of teaching to elicit children's prior understanding, you may decide to develop your questioning skills. Initially, this change may make you feel self-conscious and children may not immediately respond to the change in your questions. It is only when the different questioning mechanisms are performed with fluency that the benefits of the change are likely to be seen.

The body of knowledge to make a change is broad. It is not simply gaining a deeper knowledge of mathematics and a repertoire of teaching 'tips'. As can be seen from the discussion about beliefs, subject knowledge in relation to teaching mathematics is more than having mathematics qualifications. What is meant by the term will be explored now.

Subject knowledge

The term subject knowledge is used widely in teacher training programmes. It is used directly and indirectly throughout the Teachers' Standards which are effective from 1 September 2012 (DfE, 2011b). Students are to:

- have 'strong subject knowledge'
- 'demonstrate good subject and curriculum knowledge'
- 'have a secure knowledge of the relevant subject(s) and curriculum areas'
- 'demonstrate a critical understanding of developments in the subject and curriculum areas'

- 'impart knowledge'
- 'foster and maintain pupils' interest in the subject'.

It is hard to categorise aspects of mathematics subject knowledge as they are interrelated but three broad areas will be used.

1. *Curricular knowledge* – the mathematical content to be taught and learnt by children.
2. *Background knowledge* – the knowledge teachers require, mathematical skills and understanding, the underpinning values and philosophy.
3. *Pedagogical knowledge* – as defined as the art of teaching, *how* mathematics is taught or learned, approaches to teaching and learning.

Over the last ten years much of the pedagogical as well as the curricular knowledge has been based on national initiatives (National Numeracy Strategy (NNS) (DfEE, 1999a) and the Primary National Strategy (PNS) (DfES, 2006)). Although the status of such documents is only advisory, they were adopted in the majority of schools. While their aims of improving teaching and learning of the core subjects were laudable, they have taken many decisions regarding how to teach mathematics away from teachers. Such disempowerment has been alluded to in recent government statements about the future National Curriculum. For example, in the Schools White Paper *The Importance of Teaching* (DfE, 2011a), it is stated that the paper sets out 'school-led school improvement replacing top-down initiatives'.

As can be seen from Askew's research (Askew et al., 1997), effective teachers tended to be those who were able to make connections. The ability to make connections comes beyond the knowledge of the curriculum requirements as identified in the National Curriculum (DfEE, 1999b). It requires personal background knowledge of these connections as well as teaching and learning strategies that support making connections such as the choice of examples and knowledge of common misconceptions.

Pedagogical knowledge starts on teaching programmes in relation to school-based work. Ideas are introduced and trialled and adapted to suit the context with increasing skill. Pedagogical knowledge develops over time leading to an increasing repertoire of experience from which to draw in terms of activities and strategies as well as contexts. As a result, background understanding of mathematics evolves and knowledge deepens. Askew's research cited earlier was with experienced teachers. It is unlikely that someone in their Newly Qualified Teacher (NQT) year is going to be a highly effective teacher of mathematics but it is possible that such a person

can have a vision of where they might like to be at a certain stage in the future.

Some people enter teaching with *background* mathematics knowledge but, as can be seen from the research, many believe they do not. Teaching programmes are aimed at equipping teachers with *pedagogical* and *curricular* knowledge. Problems occur when teachers do not have sufficient background knowledge to understand or question what they are being taught. It is at this point that negative experiences and beliefs can re-emerge, often long hidden. Having knowledge without understanding is a significant factor if this knowledge is to be used to help children understand.

You may like to think about the three types of subject knowledge – curricular, background and pedagogical – and judge where your understanding lies. Sometimes it is helpful to view development as part of a continuum – where you are now, where you hope to be next year, e.g. at the beginning of your NQT year or in two years' time. You need to consider *curricular* knowledge in terms of what is taught and how it fits into the whole curriculum. *Background* subject knowledge relates to your ability in mathematics in terms of doing *and* understanding what you are doing. It involves your beliefs and attitude towards mathematics and your ability to enthuse children in their engagement with the subject. *Pedagogical* knowledge can be seen as relating to using a range of teaching and learning strategies specific to mathematics, planning suitable activities and making connections between mathematical topics.

Summary

This chapter has discussed research related to effective teaching in mathematics. It has outlined the importance of a teacher's beliefs in becoming an effective teacher of mathematics. An overview of the knowledge that a teacher requires to teach effectively has been outlined briefly in terms of three broad categories:

- curricular
- background
- pedagogical.

Conclusion

It would seem that mathematics teaching is now at a crossroads. The findings of the Williams Report (2008) strongly echo those of previous research.

Twenty-six years on from the Cockcroft Report, and despite a range of initiatives such as the implementation of the National Numeracy Strategy in 1998, little has changed. Maybe this is because the emphasis has been on curriculum content (*curricular subject knowledge*) and teaching methods (*pedagogical subject knowledge*) without addressing the more difficult and sensitive issues of teacher confidence, competence and philosophical perspective (*background subject knowledge*), as well as national inertia. As Williams states:

> . . . the United Kingdom is still one of the few advanced nations where it is socially acceptable . . . to profess an inability to cope with the subject. (Williams, 2008: 3)

Although resolving such a self-perpetuating attitude is not the sole responsibility of initial teacher education, teachers all play a fundamental part in changing such negative attitudes, not just for their own learning and practice but for future generations.

This chapter has introduced a number of issues that will be explored in later chapters. The generalist nature of teaching and subject knowledge will be discussed in Chapter 4 concerning what teachers should know. International comparisons will be discussed in detail in Chapter 8 which focuses on what can be learnt from other countries. The next chapter explores the subject of mathematics and how it has evolved into the subject being taught in schools today.

Review questions

Earlier in this chapter you were asked to consider your feelings about mathematics. It may be useful now to think about your initial reaction in order to understand the relationship between how you scored yourself on the scale at the beginning of this chapter and your beliefs about mathematics and from where they arose. You may want to categorise them under the following areas:

- Your immediate reaction to mathematics.
- Possible reasons for your response.
- The implications of your reaction to your role as a teacher of primary aged children.
- Using your response to drafting an action plan with the aim of becoming an effective teacher of mathematics.

Try to be honest with yourself and write down what you believe rather than what you think you ought to believe.

Models can be a helpful starting point in the process of reflection. Here is an example in response to the issues above but you may prefer a more discursive or diagrammatic style when you have read Appendix 2 on 'reflection'.

Journal entry 1

I love maths. I always enjoyed it at school more than English as I felt I knew if it was right or wrong. I used to try really hard writing stories and poems but never really got good marks, just comments like a 'good attempt' or 'try to write more descriptively'. I thought I had! At least with maths you know if you could do it or not.

Probably I like maths because I never saw it as a problem subject. My mother liked arithmetic. I can't remember her teaching me but she used it a lot when she was a school secretary. My father was a salesman and he would often get me to add up his sales in the evening. I do remember him showing me a quick way by making up to ten as I added. My grandmother taught me card games like 'Sevens'.

When I went to sixth form I did not choose to take maths A level because it was a time when you either chose to do sciences or arts. I didn't like the sciences that much so I chose arts subjects. However, when I trained to teach I chose to study sociology as it was different to work I had done at school. As a second subject I had to take one that could be taught in the age range I had chosen. As this was 5–11 I chose maths. The maths course was very practical – use of lots of resources with the emphasis on teaching children to understand rather than learn by rote – very different to the way I had been taught at primary school.

My interest in maths has continued throughout my time as a teacher. I have attended maths courses and I have been a maths coordinator in a number of schools. Maths is still something I do outside work. For example I often find myself estimating and calculating my time when I am swimming and I like planning mathematical designs for patchwork.

Another factor that can contribute to confidence and competence in mathematics can be the use and practice of mathematical skills on a day-to-day basis. The fact that you are reading this book means that you are likely to read, talk and write in some form on a daily basis but this may not be the situation for the complementary mathematical skills. Use and application are essential if skills are to be retained effectively. Skills can get lost when they have not been fully understood and, without use and application, this understanding can become more confused. Skills can usually be broken down into three categories

- those that you have used recently and of which you have secure understanding;
- those that you have not used recently but of which you have secure understanding;
- those you have been exposed to but do not really remember or understand.

Carry out the following activity to identify how frequently you use your mathematical skills. Note how your use of mathematical skills relates to your beliefs about mathematics.

Write down when you used any form of mathematics in

- the last 24 hours?
- the last week?

Write down when you used English in

- the last 24 hours?
- the last week?

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Websites

OECD Programme for International Student Assessment (PISA): http://www.pisa.oecd.org.

Trends in International Mathematics and Science Study (TIMSS): http://timss.bc.edu/timss2007/index.html.

Further reading

You may want to develop your understanding of mathematics anxiety. Here are some references to explore further.

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