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Introduction to the Characteristics of Number Sense

To achieve in mathematics, students must acquire a good sense of numbers early in their academic career.

Bradley S. Witzel

Introduction

A young boy and his father visit the beach for the first time. They leave their hotel, lay down their towels in the sand, and then run to the water. Immediately, they fall in love with the waves. They splash wildly among them, riding the surf into shore and then running out as far as the waves crest. They do this back and forth for about an hour. After exhaustion, they decide to check back to where they laid down their towels. However, when they turn to the shore, nothing looks the same. In fact, the high-rise hotel is nowhere in sight. Nervous, they wonder how they got there and, more importantly, how to get back to wherever they started. What had seemed like such a fun, new, and exciting time had taken them to an area where they didn't want to be.

When math education leaders made it a priority to make math more meaningful, the excellent idea was to focus more instruction on the purpose and meaning of mathematical principles rather than memorization and practice. At first, teachers created a balance between the why and the how of mathematics. However, after some years, a few argued that the why of mathematics is more important than the how. Many listened, and changes began. Modeling was de-emphasized, accuracy was devalued, and practice was limited. To some extent, these changes worked. Student engagement increased, math class was more inviting and exciting, and, as a result, students enjoyed math more than ever before. However, students' mathematics performance was not maintained.

Over the past few decades, students in the United States, especially those who show difficulties early, have underwhelmed on international mathematics achievement assessments. Statistics from international assessments and reports suggest students from the United States lack the skills to compete in an international arena, especially in fields requiring proficiency in mathematics (National Center for Education Statistics, n.d.; National Mathematics Advisory Panel [NMAP], 2008). In the example above, the great idea of playing in the waves was fun, but the father and son lost their focus. What has been learned about student engagement and activity cannot be lost, but rigor cannot be sacrificed. We have to keep our vision on learning and achievement for all students, even—and maybe especially—those who struggle early.

Turning Around Math Achievement

Within the United States, proficiency in mathematics has grown as a concern among educators, parents, policy makers, and researchers. Success in mathematics is linked to graduation, higher education, and employment (NMAP, 2008). Over the past decade, many experts have collaborated in efforts such as the Mathematics Learning Study Committee (Kilpatrick, Swafford, & Findell, 2001) and the National Mathematics Advisory Panel (NMAP, 2008) to address educational issues pertaining to mathematics proficiency. A growing consensus among experts indicates many students require curricula that provide a better foundation in mathematical number sense and more effective instruction that is grounded in research (e.g., NMAP, 2008). In order to reach goals such as increased student graduation, competitive access to higher education, and gainful employment, the NMAP emphasized the need for student success in algebra. The NMAP, therefore, recommends that students develop aptitude in prerequisite skills to algebra prior to entering high school. For this reason, particular attention must be paid to the acquisition of the concepts and skills that together develop a student's number sense.

As a result of the overall lackluster mathematical performance, prominent federally supported documents, renewed instructional emphasis of mathematics, and a significant increase in student and school expectations, educators across the United States are changing instruction to improve the mathematics learning of students. Within this discussion, several initiatives have come to the forefront, according to Riccomini and Smith (2011):

- Emphasizing number sense at all levels, but especially in the early grade levels
- Stressing the importance of high-quality mathematics instruction for all students
- Creation and adoption of the Common Core State Standards
- Setting up before- and after-school opportunities for additional mathematics instruction for struggling students
- Renewed effort to raise the awareness of importance of parents' involvement in their child's mathematics education
- Increasing the opportunities for high-quality professional development focused on evidence-based instructional techniques and strategies in the area of mathematics

Adding to the focused efforts above, a considerable paradigm shift has taken place regarding the mechanisms and procedures to address the academic needs of students who have traditionally struggled. Initially focused on early literacy, response to intervention (RTI) has quickly expanded to mathematics (Riccomini & Smith, 2011; Riccomini & Witzel, 2010). One might call the collective attention to mathematics the "perfect instructional storm" because no one at any level, from parents to interventionists, is immune to the concerns and changes within the U.S. education system.

Textbox 1.1 Math Takes a Back Seat to Reading Even at Birth

I recently celebrated the birth of my third child. The morning that we were leaving the hospital, we were given a rather large set of material. Even though I am not a new father, I decided to thumb through the massive

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packet of information given to new mothers in my state—maybe all mothers in the United States. As I looked through the documents, I came to several brochures outlining the importance of parents helping their newborn begin to develop important literacy skills. I thought that was fantastic and quickly turned to the next set of documents hoping to see a similar set of brochures focused on parents helping their newborn child learn important early numeracy skills. Much to my dismay, but not to my surprise, there was no such brochure. As a matter of fact, the entire document was void of any information related to "things" parents could do to help foster important pieces of early number sense. I guess this is probably one of the biggest differences compared to other countries—that mathematics takes a back seat to most everything else in the United States.

To make a change in education, it is important to start at the beginning. This book focuses on the construct of mathematical number sense. In this chapter, we discuss the definitions of number sense and the more prevalent core elements within and across the definitions. We conclude with number sense as outlined in the Common Core State Standards (CCSS).

Number Sense Defined

The term *number sense* means different things to different people. In a recent meeting, teachers revealed several perceived definitions of number sense covering anything from numeral recognition to conceptual understandings of complex problem solving. In order to explore instructional aspects of teaching number sense, everyone must agree on the definition. It is important to note that number sense is not necessarily a new construct in mathematics, but rather one that is only recently receiving a great deal of attention. With this renewed attention, many educators and researchers are looking for better and more expansive descriptions. Regardless of the source and definition, common elements emerge across definitions.

Holistically, Gersten and Chard (1999) described number sense as follows:

> Number sense is an emerging construct (Berch, 1998) that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons. (p. 18)

To operationalize number sense, the NMAP (2008) provided the following description:

> In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities; . . .
>
> - [T]his more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms. . . .
> - [P]oor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable. (p. 27)

Geary, Hoard, and Hamson (1999) attempted to explore more specific details of number sense and reported that students with disabilities begin to demonstrate difficulties with counting knowledge, number naming and writing, and retrieval of facts as early as first grade when compared to students without disabilities. Suggesting that these early misconceptions and difficulties in the foundational pieces of basic mathematics will likely affect students' future mathematical outcomes, they labeled these problems as important pieces of number sense. The suggestion by Gersten and Chard (1999) was later confirmed by researchers who examined growth trajectories of kindergarten students with disabilities (Morgan, Farkas, & Wu, 2009). They concluded that 70% of students in the lowest 10th percentile at the start and finish of kindergarten were likely to remain in the same percentile five years later. Clearly, students who fail to acquire the pieces that make up number sense as early as kindergarten are at greatest risk for failure in mathematics in the long term.

Textbox 1.2 A Checklist of a Numerically Powerful Child

1. Develops ***meaning*** for numbers and operations.

   Developing meaning of numbers and operations related to real life contexts is crucial for the subsequent three categories. Since children are exposed to meaning of numbers very early through daily living activities, it is important for formal mathematics instruction to continue to develop quantity, associations between and within the four operations as well as recognizing and generating differing representations of numbers. This category forms the foundation of future number sense development.

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2. Looks for ***relationships*** among numbers and operations.

   In this category, formal knowledge of numbers and operations begins to take shape by extending the meaning developed initially. Students are able to decompose numbers in varying forms ($2 + 6 = 8$; $2 + 4 + 2 = 8$; $8 - 6 = 2$) and begin to recognize how numbers and operations are interconnected. The interconnectedness of mathematics is essential for the correct application of strategies and procedures.

3. ***Understands*** computation strategies and uses them appropriately and efficiently.

   In this category, students begin to apply formal rules and procedures with an understanding of why a specific procedure or algorithm is appropriate. Students pay more attention to accuracy and precision in their answers as well as demonstrating flexibility with various strategies based on the context of the problem. Estimation becomes a crucial piece in the development of the understanding of efficient and correct strategies.

4. ***Makes sense*** of numerical and quantitative situations.

   In this category, students begin to develop and seek out calculations and relationships that make sense in the context of real life problems. The "making sense" of mathematics in this category is based on the execution of strategies and algorithms resulting in precise answers that are context dependent. Students' development with the ability to make sense of answers in the real world is the cornerstone of the application of mathematics, the ultimate goal of mathematics teaching.

Source: Adapted from Charles & Lobato (2000).

The Common Core State Standards

While every state has enacted new policies to improve mathematics education, almost every state has adopted a common list of standards intended to drive instruction for years to come. The Common Core State Standards (CCSS) were recently created as a set of sequential grade-level expectations meant to replace individual state standards. Many of the states and schools that have already begun teaching the CCSS report that expectations are more rigorous with clearer progressions across grade levels. Proponents of these new benchmarks see them as a means of improving the academic achievement of U.S. students. Those who live in states that have not fully adopted the CCSS should still expect an impact from these standards because of changes in textbook content to satisfy the majority of states that have adopted the standards. For more information on the CCSS, see www.corestandards.org.

Despite the large number of states involved and the number of people affected by this shift in standards and increased expectations, many are still concerned about the CCSS. Will state assessments be validated in time? How will current instructional materials be adapted to implement the CCSS? Who is helping prepare administrators, instructional coaches, teachers, interventionists, and parents for the changes? Many questions need to be answered during this process of change. Many educators may view these changes as merely another passing fad, theoretical model, or "best practice." Instead, these changes represent a substantive alteration in curricula and instructional emphasis. The CCSS will likely result in increased instructional rigor and a more focused approach to number sense.

The CCSS and Elements of Number Sense

Students must be taught differently than before, at least for younger grade levels. For example, one notable change from the CCSS splits the previously developed NCTM operations standard into (a) operations and algebraic thinking and (b) operations and the base-10 language. Operations designed to lead to algebraic facility allow children to learn the relationship between addition and subtraction using notation other than whole numbers (e.g., $7 + n = 15$; $15 - n = 7$). The operations and base-10 standard strand focus on operational facility based on place value that reinforces the presentation of numbers as multiples of 10. These examples are only part of the changes suggested by the CCSS that underlie the increased emphasis on number sense.

Of the 22 kindergarten common core standards, 14 can be directly linked to elements of number sense. Shown in Table 1.1 is a trajectory of the number sense elements included in the CCSS across Grades K–3. Counting and numeral and number recognition begin early, and proficiency is expected by the end of Grade 1. Magnitude comparisons with single-digit comparisons begin in

Table 1.1 Trajectory of Number Sense Elements Included in the CCSS Across Grades K–3

Number Sense Component	*Kindergarten Common Core Standard*	*First-Grade Common Core Standard*	*Second-Grade Common Core Standard*	*Third-Grade Common Core Standard*
Numeral and Number Recognition	→	→		
Magnitude Comparisons (place value)	→	→	→	→
Counting Principles	→	→		Begins with fractions
Fact Fluency	→	→	→	→
Math Language	→	→	→	→

kindergarten, but understanding and applying place value begin soon afterward and continue as part of solving operations. Different from years past, fact fluency is emphasized early and often. From addition and subtraction to multiplication and division, math language is implied throughout the grades. However, even in kindergarten, problem solving begins in a concrete manner. By first grade, word problems begin using addition.

Book Content

The elements of number sense that we emphasize in this book include quantity and cardinality (Chapter 4), numeral and number recognition (Chapter 4), strategic counting (Chapter 4), magnitude comparisons (Chapters 4 and 5), fact fluency (Chapters 4 through 6), multiplication and division (Chapter 6), algebraic concepts (Chapter 7), math language and problem solving (Chapter 8), and vocabulary (Chapter 9). In addition, we address mastery learning (Chapter 2), assessment within number sense (Chapter 3), and how to combine literacy in math and integrate math across course content (Chapter 10).

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