appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

MP8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. For example, upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (Text from Standards for Mathematical Practice,)

Each of the 5 Principles is associated with several of the Practices, and each strategy within the Principles includes a list of the MPs connected with it. Figure 2.2 at the end of this chapter also helps you cross-reference the Practices with the strategies and Principles.

THE 5 PRINCIPLES OF THE MODERN MATHEMATICS CLASSROOM

The eight Standards of Mathematical Practice describe performances and habits of mind that teachers want students to exhibit. Although they can and should be explicitly taught, if this explicit instruction is done in isolation, the Practices suffer the same fate as the algorithms and definitions we teach: becoming just one more thing that students have to remember, but that they will have difficulty applying and transferring to new situations.

The Practices will thrive, however, if students learn them in an environment designed to support them. Through two decades of teaching and supervising elementary math, I have developed a framework for such an environment, which I call the 5 Principles of the Modern Mathematics Classroom. A classroom culture built on this framework allows your students to grow into mathematical thinkers and sophisticated problem solvers. A modern mathematics classroom incorporates all 5 of these Principles:

Conjecture. In a traditional mathematics classroom, the primary goal is for students to get the right answers to questions and exercises. In a modern mathematics classroom, conjecture is encouraged, students ask most of the questions, and the answer to a question is very often another question. Inquiry is important, as is problem finding.

Communication. In a traditional classroom, communication is primarily one-way: the teacher explaining a procedure or algorithm to students. In a modern mathematics classroom, students must learn to communicate frequently about problems and how they solve them. They focus on vocabulary, writing, and metacognition. The essence of mathematics communication is the formulation and support of mathematical arguments.

Collaboration. In a traditional classroom, students work alone, and the emphasis is on an individual's skill fluency. Modern mathematics classrooms are all about the "we." Group work is far more prevalent than individual work, and students are encouraged to share ideas, answers, and to ask for help. Although there is a time for individual performance, in a problem-solving culture, the other students are cheerleaders instead of competitors.

Chaos. In a traditional classroom, neatness and order rule the day. Students must learn a procedure and then replicate it with mechanical precision. In the modern mathematics classroom, real problems require experimentation, false starts, mistakes, and corrections—sometimes over and over again. Although the term "Chaos" may sound sketchy, it simply encapsulates the idea that real math work is messy.

Celebration. In a traditional classroom, recognition is given for right answers and high grades. In a modern mathematics classroom, anything that leads toward a solution is celebrated: finding one small step of a complicated problem, thinking of an innovative approach, even if it doesn't pan out, or making a spectacular mistake and asking for help. Effort is rewarded over achievement, reflecting Carol Dweck's (2006) research on growth and fixed mindsets.