

## Chapter 2

# Implementing Effective Teaching

*Dear Coach,*

Here is support for you as you work with teachers on implementing effective teaching.

In the **COACH'S DIGEST ...**

**Overview:** To review effective teaching and/or download and share with teachers.

**Coaching Considerations for Professional Learning:** Ideas for how to support a teacher or a group of teachers in learning about effective teaching, Mathematical Practices, and *Shifts in Classroom Practice*.

**Coaching Lessons From the Field:** Story or completed tool from mathematics coaches related to ideas from this chapter.

**Coaching Questions for Discussion:** Menu of prompts for professional learning or one-on-one coaching about the *Shifts* and Mathematical Practices.

**Where to Learn More:** Articles, books, and online resources for you and your teachers!

In the **COACH'S TOOLKIT ...**

Eleven tools focused on effective teaching, for professional learning or coaching cycles.

### Coach's Digest

In the Coach's Digest, we begin with an overview of effective teaching, written to teachers (and to you, the coach). As you read the Overview, the following questions might help you reflect on this topic in terms of your role as a mathematics coach:

- Which of the Effective Mathematics Teaching Practices (and related *Shifts in Classroom Practice*) do you anticipate your teachers will want to prioritize?
- Which ones do you think need to be prioritized?

## Overview of Implementing Effective Teaching



To download the Chapter 2 Overview, go to [resources.corwin.com/mathematicskoaching](https://resources.corwin.com/mathematicskoaching).

Every student must have access to a high-quality mathematics learning experience. What does a high-quality learning experience look like? The answer is both simple and complex. Put simply, it is a daily experience in which a major focus is on mathematical proficiency, and therefore, the development of Mathematical Practices supersedes and interweaves with the content goals of a lesson. In other words, it matters *at least as much* that students can reason abstractly and quantitatively as that they can find the sum of two values. Mathematical proficiencies or processes are described effectively in the Mathematical Practices (NGA & CCSSO, 2010). Too often, the discussion of what students need to learn gets sidetracked with a dichotomy-type focus—for example, some argue that students need to understand what they are doing, while others argue they need to be efficient at using skills. The research is solid, however, in asserting that both strong conceptual understanding and procedural skills are absolutely essential in developing mathematical proficiency. The Mathematical Practices encompass this inclusive and comprehensive focus on mathematics; therefore, they must be a primary focus in any discussion about what students need to know and be able to do.

Developing mathematical proficiency (i.e., the Mathematical Practices) can be accomplished when teachers require that students engage in such practices as the way in which they learn about mathematics. There is significant evidence pointing toward Teaching Practices that support the development of mathematical proficiency, and these are comprehensively described in *Principles to Actions: Ensuring Mathematical Success for All* as Effective Mathematics Teaching Practices (see Figure 2.1).

**Figure 2.1**

### Effective Mathematics Teaching Practices From *Principles to Actions: Ensuring Mathematical Success for All*

<b>Effective Mathematics Teaching Practices</b>	
	Effective teaching of mathematics . . .
Establish mathematics goals to focus learning.	. . . establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem-solving.	. . . engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allows multiple entry points and varied solution strategies.
Use and connect mathematical representations.	. . . engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem-solving.
Facilitate meaningful mathematical discourse.	. . . facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions.	. . . uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding.	. . . builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Figure 2.1**

	Effective teaching of mathematics . . .
Support productive struggle in learning mathematics.	. . . consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking.	. . . uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Source: National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: Author.

Some of the Teaching Practices have clear and direct connections to Mathematical Practices. For example, supporting productive struggle (Teaching Practice) will support students' perseverance (Mathematical Practice). The better a teacher becomes at improving his or her ability to support productive struggle, the more a student has opportunities to make sense of mathematics and persevere. The effectiveness of a teacher in supporting productive struggle can be visualized as a continuum, as illustrated in Figure 2.2.

**Figure 2.2****Example of a Shift in Classroom Practice****Shift 7: From *mathematics-made-easy* toward *mathematics-takes-time***

Teacher presents mathematics in small chunks so that students reach solutions quickly.



Teacher questions, encourages, provides time, and explicitly states the value of grappling with mathematical tasks, making multiple attempts, and learning from mistakes.

Notice that the goal is to move Teaching Practices toward the right end of the continuum (which capture the essence of one of the NCTM Effective Mathematics Teaching Practices).

The connections between Mathematical Practices for students and Teaching Practices are actually not as straightforward or one-to-one as this example. Other *Shifts in Classroom Practice* can also impact a student's ability to make sense of and persevere in solving problems, for example. And teachers may not be focused specifically on a *Shift* or related teaching practice; instead, they may be zooming in on one topic—a Focus Zone such as formative assessment—and with this focus, their Teaching Practices are shifting to the right, thereby increasing students' opportunities to learn. This relationship is illustrated in the Leading for Mathematical Proficiency (LMP) Framework in Figure 2.3.

Professional learning can go in a myriad of directions. Attending a conference or reading a professional journal provides opportunities to explore many different zones. This can feel eclectic and unfocused, even overwhelming, as there are so many teaching ideas and **instructional strategies** that might support students. A way to bring cohesion to professional learning is to use this Framework. That means that whatever the professional learning focus might be, it is connected to the eight Effective Mathematics Teaching Practices, noticing professional growth along the *Shifts in Classroom Practice*, and ultimately, having data to support that this Practice or focus had an impact on students' emerging mathematical proficiency.



## Coaching Considerations for Professional Learning

One of the greatest challenges for a mathematics coach is to work with teachers to identify a focus for professional growth. This may be a short-term target for a coaching cycle or a long-term goal to be developed over the entire year. These goals will vary based on teachers' years of experience, as well as how receptive they are and how long you have been working with them. It is important to get a good feel for what strengths and needs a teacher has and to base the first experiences on this. Here are some ideas for how to engage in this work.

1. **Explore the Mathematical Practices.** There is a difference between having a general awareness of the standards versus having a deep understanding of what they mean. Time is well spent in providing opportunities for teachers to read and discuss aspects of the standards, considering what is new in terms of content and what this means for teaching. In particular, the Mathematical Practices require significant attention. A single Practice could be the focus of a workshop with teachers or a planning conversation. Teachers may identify those that they feel are (1) most difficult for them to get students to do and/or (2) most important for students at their grade or course level. This can be a good place to start. We provide a collection of professional development tools in Chapter 12 related to the Mathematical Practices. Additionally, several of the tools focus on any or all of the Mathematical Practices (see Tools 2.4, 2.6, and 2.9), and other tools in the Coach's Toolkit integrate the Mathematical Practices with effective planning and teaching (see Tools 2.3 and 2.5).
2. **Explore Shifts in Classroom Practice.** Like the Mathematical Practices, each of the *Shifts* is complex and must first be understood. Brainstorming teacher actions (things a teacher might *say* and things a teacher might *do*) at either end of the continuum, and in the middle, can help to solidify the focus of the *Shift* (see Tool 12.11). Teachers can self-assess where they feel they are with respect to each *Shift*, and this can begin a conversation about which *Shift* they feel will best support student engagement with the Mathematical Practices (see Tool 2.1). Additionally, several tools in the Coach's Toolkit focus on *Shifts in Classroom Practice* (see Tools 2.4, 2.8, and 2.10).
3. **Review video or print cases.** **Cases** provide an opportunity to look into a classroom of someone unknown, and this helps to develop an understanding of what it means to analyze, discuss, and adapt our teaching skills. It also provides concrete examples of the Mathematical Practices and the *Shifts*. Print cases provide the advantage of identifying specific teacher moves that result in particular student actions; video cases provide the opportunity to see a classroom in action. In the resource section that follows, we share some sites for finding cases, and new cases and resources continue to emerge.

### Coaching Lessons From the Field

We have used the LMP Framework and the *Shifts in Classroom Practice* in some of our projects. The LMP Framework afforded a lens for leadership teams to examine what support teachers needed to be able to provide opportunities for students to interact with the Mathematical Practices.

The *Shifts* provided an opportunity for teachers, principals, or leadership teams to

reflect on where they were as individuals or collectively as a school. This often set the stage for schools to consider their yearlong school goals and plans. The selected *Shift* would be written into the plan with details about how they could push that classroom practice further to the right on the continuum. Focusing on the *Shifts* with leadership teams helped to build continuity within the project,

(Continued)

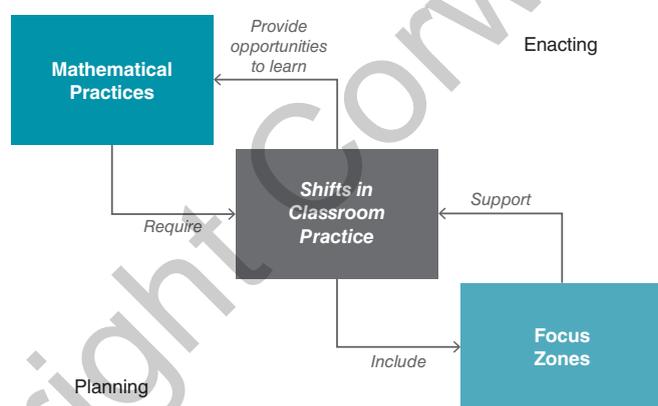


### Questions Related to Implementing Effective Teaching

1. You may have noticed that the CCSS content standards are stated concisely and do not refer to other mathematics that is needed or precedes the stated standard. As you look at one standard, what do you do to determine how this individual standard connects to the other content the student has learned or will be learning?
2. Do you think that each of the Mathematical Practices is created equal? In other words, in your grade or course level which of these Mathematical Practices might be more applicable and/or require more explicit development on your part?
3. Are all the Teaching Practices and/or *Shifts in Classroom Practice* created equal? In other words, in your grade or course level which of the *Shifts* might be more applicable to your classroom?

### Questions Related to the LMP Framework

At the heart of the Framework are the *Shifts in Classroom Practice*. Each is listed here with possible discussion questions.



#### **Shift 1: From *stating-a-standard* toward *communicating expectations for learning***

- What are some statements you might use to communicate expectations for learning?
- When and how might you communicate lesson goals in ways that make sense to students?

#### **Shift 2: From *routine tasks* toward *reasoning tasks***

- What might be some ways to turn a routine task into a reasoning task?
- What might get in the way of students reasoning and thinking for themselves?
- How can you communicate to students that their reasoning is valued and important?

#### **Shift 3: From *teaching about representations* toward *teaching through representations***

- What kind of support might students need in order to determine for themselves the correctness of their thinking?

- What are some ways that you make connections between mathematical content?
- How might you structure a lesson to use representations to strengthen mathematical understanding?

**Shift 4: From *show-and-tell* toward *share-and-compare***

- What might be some benefits for students in making connections between mathematical ideas? For you, the teacher?
- What kind of supports might students need in order to learn how to explain their thinking to others?

**Shift 5: From *questions that seek expected answers* toward *questions that illuminate and deepen student understanding***

- What kinds of instructional strategies might you use to include high-level thinking and complex tasks?
- In what ways might you ensure that all students have access and opportunity to engage in solving problems?
- What kind of support might students need in order to determine for themselves the correctness of their thinking?

**Shift 6: From *teaching so that students replicate procedures* toward *teaching so that students select efficient strategies***

- How might you help students connect conceptual and procedural knowledge?
- How might you incorporate flexibility and strategy selection into lessons?
- How might you structure a lesson or task so that students have opportunities to contrast strategies?

**Shift 7: From *mathematics-made-easy* toward *mathematics-takes-time***

- How might you provide structure or guidance for a task without giving away exactly how to do it?
- What might be some benefits of allowing students to engage in productive struggle?
- What are some ways that you challenge students to persevere on mathematical tasks?

**Shift 8: From *looking at correct answers* toward *looking for students' thinking***

- How do you decide what representations, solution strategies, or concepts you will be looking for as you observe students working?
- What might be some ways you might gather data on student thinking during a lesson? As a written work sample?

**Connecting *Shifts* to Mathematical Practices**

- As you enact the ideas of Effective Mathematics Teaching, what do you see as changes in students' opportunities to learn?
- As you enact the ideas of Effective Mathematics Teaching, what do you see as ways students are developing proficiency with the Mathematical Practices?

# Where to Learn More

## Books

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National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: Author.

*This most recent of NCTM's standards documents provides excellent, pragmatic, and research-based quick reads about each Teaching Practice, as well as Principles of Effective Teaching.*

National Council of Teachers of Mathematics (NCTM). (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices*. Reston, VA: Author.

*Like the previous book, these books (K–5, 6–8 and 9–12) are an elaboration of PtA. Each book focuses on the Teaching Practices, including vignettes, tools for analyzing teaching and learning, and ideas for how to implement the ideas in the classroom. Good resources for PLCs, a book study, or other professional learning.*

Spangler, D. A., & Wanko, J. J. (Eds.). (2017). *Enhancing Classroom Practice With Research Behind Principles to Actions*. Reston, VA: NCTM.

*This is one of several resources that followed the publication of PtA in order to provide more support to teachers and teacher leaders. This book has one chapter per Teaching Practice or Principle—each one provides a strong research base and pragmatic strategies for implementing that particular practice or principle. A good book for your own support as a coach or to study any one of these topics as a group.*

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). *Elementary and Middle School Mathematics: Teaching Developmentally* (10th ed.). Boston, MA: Pearson. [And the related *Teaching Student Centered Mathematics* three-book series, 2018.]

*This comprehensive K–8 book is a go-to reference book. Part I of the book offers many good ideas and resources related to the standards and the Shifts. Part II addresses content, focusing on what is important and how to teach it. This book was also adapted into grade-band books (K–2, 3–5, 6–8)—for example, *Teaching Student-Centered Mathematics: Grades 6–8* (Van de Walle, Lovin, Bay-Williams, & Karp, 2018). Each grade-band book provides additional activities and more elaboration on content for that grade band.*

## Articles

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Billings, E. M. H. (2017). “SMP That Help Foster Algebraic Thinking.” *Teaching Children Mathematics*, 23(8), 476–483.

*Algebraic thinking is something we can infuse into much of mathematics. This article provides instructional strategies and excellent question frames to help teachers infuse algebraic thinking, and therefore the Mathematical Practices. While this was written for K–5 teachers, it certainly applies across K–12.*

Bleiler, S. K., Baxter, W. A., Stephens, D. C., & Barlow, A. T. (2015). “Constructing Meaning: Standards for Mathematical Practice.” *Teaching Children Mathematics*, 21(6), 337–344.

*Wondering how to dig deeper into the meaning of the Mathematical Practices? This article describes tasks and activities teachers did in order to really make sense of the subtle nuances and complex ideas within a Mathematical Practice.*

Bostic, J. D., & Matney, G. T. (2014, Summer). "Role-Playing the Standards for Mathematical Practice: A Professional Development Tool." *NCSM Journal*, 3–10.

*This article offers enough guidance that you can implement this professional learning with your teachers and provides great ideas for seeing what the Mathematical Practices look like in action.*

Mateas, V. (2016). "Debunking the Myths About the Standards for Mathematical Practice." *Mathematics Teaching in the Middle School*, 22(2), 92–99.

*These myths are likely to resonate with you and to help you think about the Mathematical Practices, and this article can prompt rich discussion about how to effectively incorporate Mathematical Practices in the classroom. The examples are very helpful in illuminating key ideas about the Mathematical Practices. Access online at <http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/2016/Vol22/Issue2/Debunking-Myths-about-the-Standards-for-Mathematical-Practice>.*

## Online Resources

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### **Illustrating the Standards for Mathematical Practice**

<http://www.mathedleadership.org/ccss/itp/index.html>

*Offering ready-to-use PowerPoints and activity pages focused on the Mathematical Practices, this site can be very useful for helping teachers see the Mathematical Practices as they connect to content at their grade level or for their course.*

### **Illustrative Mathematics**

<https://www.illustrativemathematics.org>

*Developed to provide support for teachers implementing CCSS, this site has a wealth of resources. You can select Mathematical Practices, identify the particular practice of interest, and find various vignettes and videos. They have also developed free curriculum that follows the progressions in the standards.*

### **Implementing the Mathematical Practice Standards (EDC)**

<http://mathpractices.edc.org>

*The Education Development Center (EDC) has a collection of resources for the mathematics coach for each of the Mathematical Practices. Examples include a mathematics task, connections to content standards, student dialogue for that task, teacher reflection questions, and more.*

### **Inside Mathematics**

<http://www.insidemathematics.org/common-core-resources/mathematical-practice-standards>

*Video excerpts of lessons involving teachers and students across elementary, middle, and high school grade levels as they are engaged in mathematics learning experiences help to illustrate the Mathematical Practices in action. Just as with the content standards, not every lesson reflects all elements of the individual Standards for Mathematical Practice. Videos highlight the many different ways teachers may promote student implementation of the Mathematical Practices in their classrooms with their learners.*

### **NCTM Principles to Action Professional Learning Toolkit**

<http://www.nctm.org/PtAToolkit>

*Professional learning modules have been created for each of the Teaching Practices that include mathematical tasks, narrative and video cases, student work samples, and vignettes. A wealth of information for a mathematics coach.*

# Coach's Toolkit

These tools are a menu from which you can select any that make sense for your setting/context. They can be used independently or as part of a coaching cycle. You may start with the self-assessment, which can guide you in deciding which of the other tools may be most useful. If using these tools for a coaching cycle, mix and match as you like or use one of the combinations we suggest in the diagrams that follow. The tools in this chapter include instructions to the coach and the teacher. You can download copies of the tools that only have instructions for the teacher at [resources.corwin.com/mathematicskoaching](http://resources.corwin.com/mathematicskoaching).

## Self-Assess



- 2.1** *Shifts in Classroom Practice* Self-Assessment

## Gather Data



- 2.6** Mathematical Practice Look Fors  
**2.7** *Shifts in Classroom Practice*  
**2.8** Effective Teaching Look Fors

## Plan

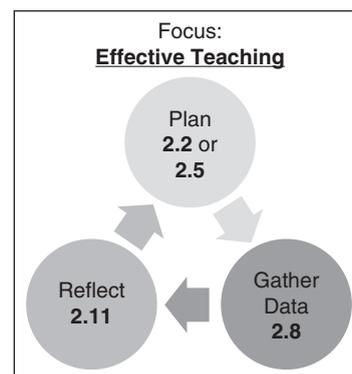
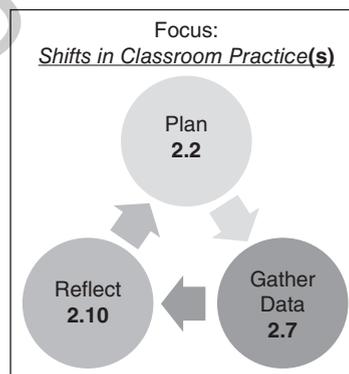
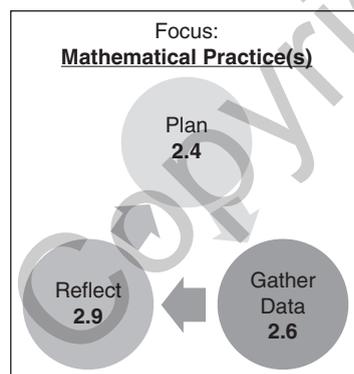


- 2.2** Essential Planning Questions for Effective Teaching  
**2.3** Practices, *Shifts*, and Zones (Oh My)  
**2.4** Mathematical Practices by Design  
**2.5** Lesson Plan Template

## Reflect



- 2.9** Noticing Mathematical Practices  
**2.10** Mapping Teaching Moves to *Shifts in Classroom Practice*  
**2.11** Effective Teaching of Mathematics



## Additional Tools in Other Chapters

More focus on Mathematical Practices and *Shifts in Classroom Practice* can be found in all the Focus Zone chapters (Chapters 3–10).



To download the coaching tools for Chapter 2 that only have instructions for the teacher, go to [resources.corwin.com/mathematicskoaching](http://resources.corwin.com/mathematicskoaching).



## 2.1 Shifts in Classroom Practice Self-Assessment

*Instructions to Coach: Ask teachers (individually or as part of a PLC activity) to self-assess where they position themselves on each of these Shifts in Classroom Practice. Use the results to focus on next steps. A one-page version of this tool without this note is available online for download.*

Instructions: Place an X along each continuum that best represents your classroom practice.

### **Shift 1: From stating-a-standard toward communicating expectations for learning**

Teacher shares broad performance goals and/or those provided in standards or curriculum documents.



Teacher creates lesson-specific learning goals and communicates these goals at critical times within the lesson to ensure students understand the lesson's purpose and what is expected of them.

### **Shift 2: From routine tasks toward reasoning tasks**

Teacher uses tasks involving recall of previously learned facts, rules, or definitions and provides students with specific strategies to follow.



Teacher uses tasks that lend themselves to multiple representations, strategies, or pathways encouraging student explanation (how) and justification (why/when) of solution strategies.

### **Shift 3: From teaching about representations toward teaching through representations**

Teacher shows students how to create a representation (e.g., a graph or picture).



Teacher uses lesson goals to determine whether to highlight particular representations or to have students select a representation; in both cases, teacher provides opportunities for students to compare different representations and how they connect to key mathematical concepts.

### **Shift 4: From show-and-tell toward share-and-compare**

Teacher has students share their answers.



Teacher creates a dynamic forum where students share, listen, honor, and critique each other's ideas to clarify and deepen mathematical understandings and language; teacher strategically invites participation in ways that facilitate mathematical connections.



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**Shift 5: From questions that seek expected answers toward questions that illuminate and deepen student understanding**

Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.

Teacher poses questions that advance student thinking, deepen students' understanding, make the mathematics more visible, provide insights into student reasoning, and promote meaningful reflection.

**Shift 6: From teaching so that students replicate procedures toward teaching so that students select efficient strategies**

Teacher approaches facts and procedures with the goal of speed and accuracy.

Teacher provides time for students to engage with mathematical problems, developing flexibility by encouraging student selection and use of efficient strategies; teacher provides opportunities for students to evaluate when a strategy is best suited for the problem at hand.

**Shift 7: From mathematics-made-easy toward mathematics-takes-time**

Teacher presents mathematics in small chunks so that students reach solutions quickly.

Teacher questions, encourages, provides time, and explicitly states the value of grappling with mathematical tasks, making multiple attempts, and learning from mistakes.

**Shift 8: From looking at correct answers toward looking for students' thinking**

Teacher attends to whether an answer or procedure is (or is not) correct.

Teacher identifies specific strategies or representations that are important to notice; strategically uses observations, student responses to questions, and written work to determine what students understand; and uses these data to inform in-the-moment discourse and future lessons.





## 2.2 Essential Planning Questions for Effective Teaching

*Instructions to Coach: This tool can be fully or partially filled out; a teacher can complete it prior to a planning conversation, or it can be the product of a planning conversation.*

Instructions: The following questions can be part of a planning conversation or collaborative lesson design opportunity focused on implementing the Teaching Practices (TPs).

Teaching Practice Planning Questions	Lesson Planning Decisions
TP#1: What should students know (content) and be able to do (content and Mathematical Practices) in this lesson, and how will I make these expectations clear to students?	
TP#2: How will the task or lesson I selected include multiple pathways and elicit student reasoning?	
TP#3: What representations need to be connected to the content of this lesson, and does this mean I will select or that students will select the representations?	
TP#4: How and when will we discuss the important mathematical ideas of the lesson?	
TP#5: What questions will I ask to help students make connections among solution strategies and among mathematical ideas?	
TP#6: How will I ensure students are understanding strategies/algorithms and learning to make good choices about when to use a particular strategy/algorithm?	
TP#7: What structures, feedback, and instructional moves can I use to ensure that students grapple with the tasks in the lesson and strive to make mathematical connections?	
TP#8: What will I have students do at the very end of the lesson that summarizes the big ideas of the lesson?	



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## 2.3 Practices, Shifts, and Zones (Oh My)

*Instructions to Coach: You can identify a starting place or ask teacher(s) to select one. As they work, focus on evidence of what it looks like in practice.*

Instructions: Identify the Mathematical Practices, Shifts, and Focus Zones that will be the target of your lesson/unit. You may start in any of the boxes, discussing how selections in one area connect to selections in the other areas.

<i>Selection and what it looks like</i>	
<p><b>Mathematical Practices</b></p> <ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>	
<p><b>Shifts in Classroom Practice</b></p> <p>...Toward...</p> <ol style="list-style-type: none"> <li>1. Communicates learning expectations.</li> <li>2. Reasoning tasks.</li> <li>3. Teaching through representations.</li> <li>4. Share-and-compare.</li> <li>5. Questions illuminate and deepen student understanding.</li> <li>6. Selecting efficient strategies.</li> <li>7. Mathematics-takes-time.</li> <li>8. Valuing students' thinking.</li> </ol>	
<p><b>Focus Zones</b></p> <ol style="list-style-type: none"> <li>1. Content Knowledge and Worthwhile Tasks</li> <li>2. Engaging Students</li> <li>3. Questioning and Discourse</li> <li>4. Formative Assessment</li> <li>5. Analyzing Student Work</li> <li>6. Differentiating Instruction for All Learners</li> <li>7. Supporting Emerging Multilingual Students</li> <li>8. Supporting Students with Special Needs</li> </ol>	



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## 2.4 Mathematical Practices by Design

*Instructions to Coach: Not all Mathematical Practices fit all lessons. And even if many are likely to be evident, it is best to focus on fewer (1–3), and these should be based on “best fit” for the learning outcome and the task selected. Discuss which MPs make sense, and then discuss design plans.*

Instructions: Highlight the Mathematical Practices and/or Look Fors that will be prominent in the lesson. Write design plans for how the selected Mathematical Practice will be developed.

Topic/Goal of Lesson: \_\_\_\_\_

<b>Mathematical Practices and Student Look Fors</b>	<b>Design Plans</b>
1. Make sense of problems and persevere in solving them. <ul style="list-style-type: none"> <li>• Analyze information (givens, constraints, relationships, goals).</li> <li>• Make conjectures and plan a solution pathway.</li> <li>• Use objects, drawings, and diagrams to solve problems.</li> <li>• Monitor progress and change course as necessary.</li> <li>• Check answers to problems and ask, “Does this make sense?”</li> </ul>	
2. Reason abstractly and quantitatively. <ul style="list-style-type: none"> <li>• Make sense of quantities and relationships in problem situations.</li> <li>• Create a coherent representation of a problem.</li> <li>• Translate from contextualized to generalized or vice versa.</li> <li>• Flexibly use properties of operations.</li> </ul>	
3. Construct viable arguments and critique the reasoning of others. <ul style="list-style-type: none"> <li>• Make conjectures and use counterexamples to build a logical progression of statements to support ideas.</li> <li>• Use definitions and previously established results.</li> <li>• Listen to or read the arguments of others.</li> <li>• Ask probing questions to other students.</li> </ul>	
4. Model with mathematics. <ul style="list-style-type: none"> <li>• Determine equation that represents a situation.</li> <li>• Illustrate mathematical relationships using diagrams, two-way tables, graphs, flowcharts, and formulas.</li> <li>• Check to see whether an answer makes sense within the context of a situation and change a model when necessary.</li> </ul>	



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<b>Mathematical Practices and Student Look Fors</b>	<b>Design Plans</b>
<p>5. Use appropriate tools strategically.</p> <ul style="list-style-type: none"> <li>• Choose tools that are appropriate for the task (e.g., manipulative, calculator, digital technology, ruler).</li> <li>• Use technological tools to visualize the results of assumptions, explore consequences, and compare predictions with data.</li> <li>• Identify relevant external math resources (digital content on a website) and use them to pose or solve problems.</li> </ul>	
<p>6. Attend to precision.</p> <ul style="list-style-type: none"> <li>• Communicate precisely, using appropriate terminology.</li> <li>• Specify units of measure and provide accurate labels on graphs.</li> <li>• Express numerical answers with appropriate degree of precision.</li> <li>• Provide carefully formulated explanations.</li> </ul>	
<p>7. Look for and make use of structure.</p> <ul style="list-style-type: none"> <li>• Notice patterns or structure, recognizing that quantities can be represented in different ways.</li> <li>• Use knowledge of properties to efficiently solve problems.</li> <li>• View complicated quantities both as single objects and as compositions of several objects.</li> </ul>	
<p>8. Look for and express regularity in repeated reasoning.</p> <ul style="list-style-type: none"> <li>• Notice repeated calculations and look for general methods and shortcuts.</li> <li>• Maintain oversight of the process while attending to the details.</li> <li>• Evaluate reasonableness of intermediate and final results.</li> </ul>	

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## 2.5 Lesson Plan Template

*Instructions to the Coach: This tool can be prepared by a teacher in advance or used in collaborative planning. Planning questions are provided within each cell. Use the "Lesson Reflections" cell to determine what data-gathering tool to use.*

Instructions: Complete this tool in connection with a specific lesson and selected task.

<p>Content standard(s):</p> <p>Objectives:</p>	<p>Mathematical Practices</p> <ul style="list-style-type: none"><li><input type="checkbox"/> 1. Make sense of problems and persevere in solving them.</li><li><input type="checkbox"/> 2. Reason abstractly and quantitatively.</li><li><input type="checkbox"/> 3. Construct viable arguments and critique the reasoning of others.</li><li><input type="checkbox"/> 4. Model with mathematics.</li><li><input type="checkbox"/> 5. Use appropriate tools strategically.</li><li><input type="checkbox"/> 6. Attend to precision.</li><li><input type="checkbox"/> 7. Look for and make use of structure.</li><li><input type="checkbox"/> 8. Look for and express regularity in repeated reasoning.</li></ul>
<p>Essential questions: <i>What questions will promote inquiry, understanding, and transfer of learning?</i></p>	<p>Assessment evidence: <i>By what criteria will "performance of understanding" be judged?</i></p>
<p>Focus task: <i>What specific mathematical activities, investigations, texts, problems, or tasks will students do in order to learn the content?</i></p>	<p>Anticipated student responses: <i>What prior knowledge or limited conceptions might students have? How might students solve the problem?</i></p>
<p>Resources: <i>What materials or resources are essential for students to successfully complete the lesson tasks or activities?</i></p>	<p>Anticipated language needs: <i>What words, phrases, or symbols may need to be explicitly discussed within the lesson?</i></p>



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Engage (set up the task): *Exactly how will I elicit prior content knowledge, connect to students' experiences, and set up the task (to ensure students understand the task without overscaffolding or funneling)?*

Explore (solve the task): *What questions might I ask individuals or small groups of students that focus on the content and Mathematical Practices?*

Connect (discuss task and related mathematical concepts): *What questions and/or activity will engage students in explaining and/or illustrating the concepts of the lesson, as well as provide formative assessment as to who learned what?*

Lesson reflections: *What questions connected to the standards and assessment evidence will I use to reflect on the effectiveness of this lesson?*





## 2.6 Mathematical Practice Look Fors

*Instructions to the Coach: You can fill out this tool during an observation and/or have the teacher complete it with what they are seeing and hearing. Or you can video or write down as much student talk as possible and map that data to this page in a reflecting conversation.*

Instructions: During a lesson, listen for student actions related to any or all of these Mathematical Practices. Note what they said or did in the examples column.

<b>Mathematical Practice</b>	<b>Student Look Fors</b>	<b>Examples</b>
1. Make sense of problems and persevere in solving them.	<ul style="list-style-type: none"><li><input type="checkbox"/> Analyze information (givens, constraints, relationships, goals).</li><li><input type="checkbox"/> Make conjectures and plan a solution pathway.</li><li><input type="checkbox"/> Use objects, drawings, and diagrams to solve problems.</li><li><input type="checkbox"/> Monitor progress and change course as necessary.</li><li><input type="checkbox"/> Check answers to problems and ask, "Does this make sense?"</li></ul>	
2. Reason abstractly and quantitatively.	<ul style="list-style-type: none"><li><input type="checkbox"/> Make sense of quantities and relationships in problem situations.</li><li><input type="checkbox"/> Create a coherent representation of a problem.</li><li><input type="checkbox"/> Translate from contextualized to generalized or vice versa.</li><li><input type="checkbox"/> Flexibly use properties of operations.</li></ul>	
3. Construct viable arguments and critique the reasoning of others.	<ul style="list-style-type: none"><li><input type="checkbox"/> Make conjectures and use counterexamples to build a logical progression of statements to support ideas.</li><li><input type="checkbox"/> Use definitions and previously established results.</li><li><input type="checkbox"/> Listen to or read the arguments of others.</li><li><input type="checkbox"/> Ask probing questions to other students.</li></ul>	
4. Model with mathematics.	<ul style="list-style-type: none"><li><input type="checkbox"/> Determine equation that represents a situation.</li><li><input type="checkbox"/> Illustrate mathematical relationships using diagrams, two-way tables, graphs, flowcharts, and formulas.</li><li><input type="checkbox"/> Check to see whether an answer makes sense within the context of a situation and change a model when necessary.</li></ul>	



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<b>Mathematical Practice</b>	<b>Student Look Fors</b>	<b>Examples</b>
5. Use appropriate tools strategically.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Choose tools that are appropriate for the task (e.g., manipulative, calculator, digital technology, ruler).</li> <li><input type="checkbox"/> Use technological tools to visualize the results of assumptions, explore consequences, and compare predictions with data.</li> <li><input type="checkbox"/> Identify relevant external math resources (digital content on a website) and use them to pose or solve problems.</li> </ul>	
6. Attend to precision.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Communicate precisely using appropriate terminology.</li> <li><input type="checkbox"/> Specify units of measure and provide accurate labels on graphs.</li> <li><input type="checkbox"/> Express numerical answers with appropriate degree of precision.</li> <li><input type="checkbox"/> Provide carefully formulated explanations.</li> </ul>	
7. Look for and make use of structure.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Notice patterns or structure, recognizing that quantities can be represented in different ways.</li> <li><input type="checkbox"/> Use knowledge of properties to efficiently solve problems.</li> <li><input type="checkbox"/> View complicated quantities both as single objects and as compositions of several objects.</li> </ul>	
8. Look for and express regularity in repeated reasoning.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Notice repeated calculations and look for general methods and shortcuts.</li> <li><input type="checkbox"/> Maintain oversight of the process while attending to the details.</li> <li><input type="checkbox"/> Evaluate reasonableness of intermediate and final results.</li> </ul>	

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## 2.7 Shifts in Classroom Practice

*Instructions to the Coach: Record each teacher action and interaction in a separate box (copy as needed). At the end of the lesson, cut out each piece of evidence and use Tool 2.10 to determine where each piece belongs on the Shift's continuum. Alternatively, you might record each event on a sticky note.*




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## 2.8 Effective Teaching Look Fors

*Instructions to the Coach: Preselect practices with a teacher and fill out this tool during an observation. Or video or write down as much teacher talk/actions as possible and map that data to this page in a reflecting conversation.*

Instructions: Select Teaching Practice(s) and record specific teacher moves or actions that demonstrate that Practice.

Teaching Practice (NCTM, 2014) Look Fors	Evidence
Establish mathematics goals to focus learning. <ul style="list-style-type: none"> <li><input type="checkbox"/> Goals are appropriate, challenging, and attainable.</li> <li><input type="checkbox"/> Goals are specific to the lesson and clear to students.</li> <li><input type="checkbox"/> Goals are connected to other mathematics.</li> <li><input type="checkbox"/> Goals are revisited throughout the lesson.</li> </ul>	
Implement tasks that promote reasoning and problem-solving. <ul style="list-style-type: none"> <li><input type="checkbox"/> Chooses engaging, high-cognitive-demand tasks with multiple solution pathways.</li> <li><input type="checkbox"/> Chooses tasks that arise from home, community, and society.</li> <li><input type="checkbox"/> Uses how, why, and when questions to prompt students to reflect on their reasoning.</li> </ul>	
Use and connect mathematical representations. <ul style="list-style-type: none"> <li><input type="checkbox"/> Uses tasks that lend themselves to multiple representations.</li> <li><input type="checkbox"/> Selects representations that bring new mathematical insights.</li> <li><input type="checkbox"/> Gives students time to select, use, and compare representations.</li> <li><input type="checkbox"/> Connects representations to mathematics concepts.</li> </ul>	
Facilitate meaningful mathematical discourse. <ul style="list-style-type: none"> <li><input type="checkbox"/> Helps students share, listen, honor, and critique each other's ideas.</li> <li><input type="checkbox"/> Helps students consider and discuss each other's thinking.</li> <li><input type="checkbox"/> Strategically sequences and uses student responses to highlight mathematical ideas and language.</li> </ul>	



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Teaching Practice (NCTM, 2014) Look Fors	Evidence
<p>Pose purposeful questions.</p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Questions make the mathematics visible.</li> <li><input type="checkbox"/> Questions solidify and extend student thinking.</li> <li><input type="checkbox"/> Questions elicit student comparison of ideas and strategies.</li> <li><input type="checkbox"/> Strategies are used to ensure every child is thinking of answers.</li> </ul>	
<p>Build procedural fluency from conceptual understanding.</p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Gives students time to think about different ways to approach a problem.</li> <li><input type="checkbox"/> Encourages students to use their own strategies and methods.</li> <li><input type="checkbox"/> Asks students to compare different methods.</li> <li><input type="checkbox"/> Asks why a strategy is a good choice.</li> </ul>	
<p>Support productive struggle in learning mathematics.</p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Provides ample wait time.</li> <li><input type="checkbox"/> Talks about the value of making multiple attempts and persistence.</li> <li><input type="checkbox"/> Facilitates discussion on mathematical error(s), misconception(s), or struggle(s) and how to overcome them.</li> </ul>	
<p>Elicit and use evidence of student thinking.</p> <ul style="list-style-type: none"> <li><input type="checkbox"/> Identifies strategies or representations that are important to look for as evidence of student understanding.</li> <li><input type="checkbox"/> Makes just-in-time decisions based on observations, student responses to questions, and written work.</li> <li><input type="checkbox"/> Uses questions or prompts that probe, scaffold, or extend students' understanding.</li> </ul>	

Source: Previously published by Bay-Williams, J., McGatha, M., Kobett, B., and Wray, J. (2014). *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K-12*. New York, NY: Pearson Education, Inc.



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## 2.9 Noticing Mathematical Practices

*Instructions to Coach: This tool can be used as a follow-up to Tools 2.3 Practices, Shifts, and Zones (Oh My) and 2.6 Mathematical Practice Look Fors. Teachers can have it already filled out for a reflecting conversation, or you can use these as discussion prompts.*

Instructions: In reflecting on the lesson related to evidence of students engaged in using the Mathematical Practices, respond to the following questions.

1. What evidence do you see of one instructional move or *Shift* that supported the students engaging in the Mathematical Practices?
2. What aspects of the lesson (design, questions, grouping, task) do you think were particularly effective at eliciting these Mathematical Practices or Look Fors?
3. What aspect(s) of the Look Fors were not as evident in the lesson? What might be adapted in the lesson to better support the development of those Look Fors?
4. What might be one *Shift in Classroom Practice* that could be a target for future professional development that is needed in order to better engage in the identified Look Fors?



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## 2.10 Mapping Teaching Moves to *Shifts in Classroom Practice*

*Instructions to Coach:* For professional learning, assign different groups to different *Shifts* and sort evidence from a classroom teaching video (see Tool 2.7 for recording evidence). For lesson cycle, use Tool 2.7 to gather data and then ask the teacher to place each teacher move along the continuum. Or you could each identify a location and then discuss any differences.

Selected *Shift*: \_\_\_\_\_

Cut along the dashed lines. Tape together to make a long continuum (e.g., on flip chart paper). Glue, tape, or write your selected *Shift* descriptors in the empty boxes. Decide where on the continuum the evidence might fit (and why).

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## 2.11 Effective Teaching of Mathematics

*Instructions to Coach: Select any of these prompts or use them all—the first six address the Teaching Practices, and the final two are summarizing prompts.*

Instructions: Using data from a lesson, reflect on the questions below.

1. To what extent did the lesson tasks, activities, and/or discussion support the lesson objectives?
2. What representations/strategies/approaches did students use to solve problems and demonstrate their understanding? Which ones may need more attention going forward?
3. What questioning and discourse went well, and what was challenging?
4. In what ways did you see students making connections (between concepts and procedures; between representations; to previously learned concepts; to the real world)?
5. In what ways were students asked to grapple with mathematics? How effective were your strategies to support their struggle without taking away students' thinking?
6. To what extent were you able to determine whether each student learned the objectives?
7. What do you feel were the most successful aspects of this lesson?
8. What are you learning that you would like to remember when you teach this lesson again?



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