Preface

ne of the most difficult tasks a mathematics teacher faces in the teaching process is finding interesting ways to introduce new topics or developing new concepts for a class. If the textbook does not present clever developments, the teacher must search for alternatives. Obviously, it is easier simply to follow the textbook's approach, which may be dull and uninspiring for the students. As a result, the students may see mathematics as a meaningless jumble of rules presented by the teacher and imitated by the students. Students are often bored and uninterested and make little or no attempt to understand the idea being developed. They simply memorize the rule or algorithm as it is given and assume that if they can master these rules, then they will succeed on tests. The extent to which such students understand the concepts for use in later courses or for use in real-life applications is questionable.

There are a variety of ways to discover new approaches for teaching specific topics in the mathematics classroom. One might use a historical event to introduce a topic, use a physical model to concretize a new concept, use some form of recreational game or challenge to develop an idea, or simply delve into the riches of mathematics to come up with a clever alternative approach to develop a common topic in the school curriculum. None of these techniques is particularly simple. A combination of these techniques is even more difficult to find. However, new approaches to teaching old favorite topics are an essential part of the continuous growth process that every mathematics teacher should experience throughout a teaching career. It is rejuvenating! We respond to this need with this book, which is designed to provide the mathematics teacher with 101+ Great Ideas for Introducing Key Concepts in Mathematics: A Resource for Secondary School Teachers. A by-product is a desire to stimulate teachers to search for yet other techniques.

English teachers are derelict in their responsibility if they do not read books regularly. A history teacher must keep abreast of continuously changing world events to make the teaching of history relevant to students. The science teacher must be aware of new developments in science. Where does the responsibility of the mathematics teacher lie in this regard? New developments in mathematics (such as the proofs of Fermat's last theorem and the four-color map problem), although they may be of general interest to students if presented properly, have technical aspects far beyond high school students' reach. Thus, new developments in mathematics are not particularly appropriate for the mathematics classroom. However, the mathematics teacher does have a professional responsibility to read mathematics books and journals. Doing so enriches the instructional program and

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101+ Great Ideas for Introducing Key Concepts in Mathematics

results in new and more exciting ways to teach the material in the syllabus. This book should begin to motivate such explorations.

This book presents motivational ways to teach many of the key topics in the secondary school curriculum. The suggested teaching approaches are presented as though the teacher is completely unfamiliar with them as well as with their underlying mathematics. Every effort has been made to make the discussions clear and complete. The concepts and topics presented here have been chosen as representative of those most commonly taught in the secondary school curriculum.

Some mathematics teachers feel that students who reach high school should no longer require physical models to comprehend mathematical concepts. Learning theorists have found that this is not always true. Chronological age does not always match mental maturity. Many adolescents still need the comfort of a physical model when they are learning a new concept. Consequently, some of the ideas presented in this book involve a hands-on format with materials or devices used by the students and the teacher. Naturally, there is still a heavy reliance on the chalkboard, overhead projector, and dynamic geometry software programs, such as Geometer's Sketchpad—the standard "tools of the trade."

The format used in this book will enable the teacher to easily find teaching ideas for the specific topic being taught. We arranged the topics by classical subject matter (algebra, geometry, trigonometry, probability and statistics, and miscellaneous topics), yet the actual location of an individual topic may vary with different state curricula.

The topic of each unit is provided by the identifying title. Then the *objective* to be attained by implementing the unit is stated and the materials that may be required are listed. The subsequent discussion of the technique or procedure is presented with sufficient detail so that a teacher who is entirely unfamiliar with the mathematics underlying the proposed activity will have enough support to feel comfortable with the procedure.

With the exception of the last five units of this book, the units are all independent of one another. The last five units must be read in sequence.

Teachers should begin by reading the book and noting the topics that may be of special interest. These should be tried in class. Teachers should find a marked increase in the interest and comprehension of the mathematics being presented. Finally, teachers should begin to establish a personal collection of teaching ideas beyond those presented in this book, by getting inspiration from professional journals, reading mathematics books, attending lectures at professional conferences, or simply observing experienced colleagues. We hope that this book will serve as a springboard for teachers to begin collecting their own great ideas for introducing the key concepts they teach!

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